Performance Analysis of MIMO System Using Rayleigh and Rician Fading Channel

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Approval

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A thesis report submitted to East West University as a fulfillment for the B.Sc. in Electronics and Telecommunications Engineering.

Sign…………………………..

(Thesis Supervisor)

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Abstract

System performance is an important characteristic in wireless communication. The Channel quality always cannot be determined perfectly due to different aspects like the fading effects, Interference, scattering of EM wave etc. Also due to different fading effects often the transmitted data is not recovered properly at the receiving end. In this paper we measure and analyze the performance of a wireless system in Rayleigh and Rician fading channel using Multi Input Multi Output (MIMO) and show how the capacity increases with respect to signal to noise ratio. It has been shown that by using MIMO system and Orthogonal Frequency Division Multiplexing (OFDM) technology the channel capacity and bandwidth efficiency increases. It has also been shown that data loss also minimizes using this technology.

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CHAPTER 1

Introduction

Wireless services represent a progression of technology, which is also a new era of telecommunications. Wireless system operation allows services such as long distance communications which may not be executed by wired system. Wireless communications is enjoying its rapid growth in history because of its few advantages over wired system. Comparing to any wired system wireless technology is much cheaper and installation process is a lot easier. Wireless communication is the combination of electronic products and system which interact by using electromagnetic energy carrying information between them so that they can communicate with each other. Wireless communication is used to transfer information over both short and long distances depending on requirement. The wireless industry has created a strong infrastructure for providing different services for it users. The mobile radio communications industry has grown by orders of magnitude, fueled by digital and RF circuit fabrication improvements and other miniaturization technologies which make portable radio equipment smaller, cheaper and more reliable. Mobile and fixed wireless transmission are the two types of wireless communication. In fixed transmission, transmitter and receiver remain stable while in mobile wireless transmission both the receiver and transmitter are in motion operating at different speed. So, the signal propagates through multipath and reached at the receiver at different time, delay and amplitude. Radio communication system like Universal mobile telecommunications system (UMTS) or Wireless local area network (WLAN) must required providing continuously higher data rate as smart devices nowadays are used for multitasking at the same time. Through, conventional communication systems are not been able to cope up with the pace. And that’s why a robust communication system, Multiple Input Multiple Output (MIMO) is introduced which is capable to fulfill all the criteria and gives a significant enhancement both to the data rate and channel capacity. Multiple-Input-Multiple-Output (MIMO) is a modern edge antenna technology transmitting multiple data streams to multiple receivers. MIMO let antennas at the transmitting and receiving side spatial streams at the same time. MIMO offer tremendous performance gains for wireless devices at a relatively less cost. MIMO use multiple antennas both at the receiving and transmitting side so that receiver can receive every bit of transmitted signal when link has mounted at non line of site . MIMO makes antennas work smarter by enabling them to accumulate data streams coming from different times and at different paths to accurately increase receiving end signal-capturing power. Smart antennas often use spatial diversity technology, which puts antennas to proper use. If there are more antennas than spatial streams, the additional antennas can add receiver diversity and increase range. One of the core ideas behind MIMO wireless systems is space-time signal processing where time is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas, the use of multiple antennas located at different points . Accordingly MIMO wireless systems can be viewed as a logical extension to the smart antennas that have been used for many years to improve wireless communication system. MIMO spatial multiplexing achieves that by utilizing the multiple paths and accurately using them as extra channels to carry those data. The maximum amount of data that can be carried by a radio channel is limited by the physical boundaries defined under Shannon's law. Multiple-input, multiple-output (MIMO) antenna systems are used in modern wireless standards, including in IEEE 802.11n, UMTS, LTE, and WiMAX systems. The technique supports enhanced data throughout even under conditions of interference, signal fading, and multipath. The demand for higher data rates over longer distances has been one of the primary motivations behind the development of MIMO orthogonal- frequency-division-multiplexing (OFDM) communications systems.

In this paper we have discussed about Rayleigh fading, Rician fading, flat fading, slow fading, Bit Error Rate (BER), Signal to noise ratio (SNR), Channel capacity etc. The rest of this paper is organized as follows; firstly we present different types of fading with appropriate curve and then through discussion about MIMO and its previous versions in second section. We then provided discussion about SNR, Channel capacity, Alamouti space-time block coding, Water filling model, Channel Capacity versus SNR curve. And finally the ending section gives the result and conclusion about the whole paper.

CHAPTER 2

Fading

In modern wireless communication fading is a common characteristic. Fading is caused because of both long term and short term fluctuations of the received signal compared to the transmitted signal transmitted by the transmitter. The path between the transmitter and the receiver are often affected by different obstacles, terrain causes fading. Multipath fading affects most forms of radio communications links in one form or another. Multipath fading can be detected on many signals across the frequency spectrum from the HF bands right up to microwaves and beyond. It is experienced not only by short wave radio communications where signals fade in and out over a period of time, but it is also experienced by many other forms of radio communications systems including cellular telecommunications and many other users of the VHF and UHF spectrum . Researchers have shown that multiple propagation paths or multi paths have both slow and fast aspects. The received signal for narrowband excitation is found to exhibit three scales of spatial variation such as fast fading, slow fading and range dependence [1]. Multipath fading may also cause distortion to the radio signal. As the various paths that can be taken by the signals vary in length, the signal transmitted at a particular instance will arrive at the receiver over a spread of times. This can cause problems with phase distortion and inter symbol interference (ISI) when data transmissions are made. As a result, it may be necessary to incorporate features within the radio communications system that enables the effects of these problems to be minimized.

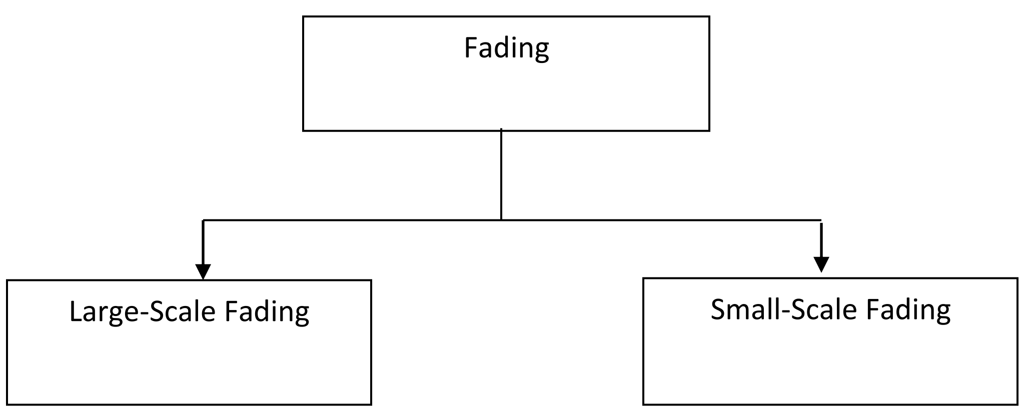

Figure-1: Two primary types of Fading

2.1 Large-Scale Fading:

Large scale fading occurs due to the shadowing effect of large sized objects. The time constant associated with the variations are very long as the receiver moves. In this fading variation occurs over very large distances. Because of the wave is shadowed or blocked by the obstacles leads to significant loss of signal strength. Generally this type of fading is independent of frequency.

2.2 Small-Scale Fading:

Small scale fading is used to describe the rapid fluctuations of amplitudes, phases, or multipath delays of a radio signal over a short period of time or travel distance [2]. Due to interference of different versions of the transmitted signal the receiver gets these different signals in different times.

In modern era fading happens due to the surrounding obstacles around the antenna and also because of the reflection, refraction and the scattering of the signal.

2.2.1 Small-Scale Fading Factors:

1. Multiple path propagation:

Due to the scattering in the channel and presence of reflecting objects results to a constantly changing environment that lessens the signal energy involving amplitude, phase, and time. For these effects multiple the transmitted signals arriving the receiving end displaced with respect to another signal in time. In multipath propagation the time required for the baseband signal to reach the receiver gets lengthy for inter symbol interference (ISI).

2. Device (receiver) speed:

If the device is moved on a fast speed there will be maximum fading. Due to the movement Doppler shift affects. If the device is moving towards the base station then there will be positive Doppler shift and if the device is moving away from the base station then there will be negative Doppler shift.

3. Speed of surrounding objects:

If the surrounding environment rapidly changes then fading occurs. But if the surrounding environment not changing and the device is moved to a short distance then there is fading due to the device not by the surrounding environment [2] .

2.2.2 Small-Scale Fading Types:

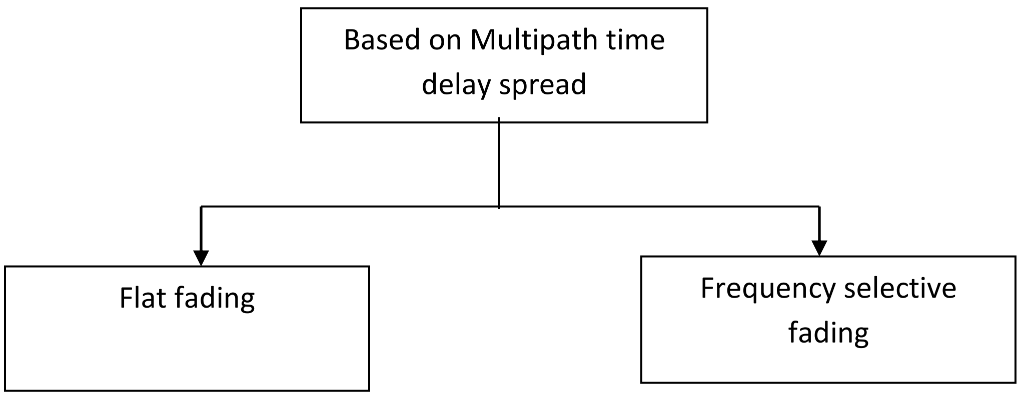

Figure-2: Fading Based on Multipath time delay spread

Flat fading:

If a channel has constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal then the channel will undergo flat fading [2]. Flat fading can also occur when the bandwidth of the transmitted signal B is smaller than the coherence bandwidth of the channel $B_m = B \times B_m$ .If the SNR drops we can see the flat fading channel.

Frequency selective fading:

If a channel has constant gain and linear phase response over a bandwidth which is smaller than the bandwidth of the transmitted signal then the channel will undergo frequency selective fading. The delay spread is also greater than the symbol period. These fading channels are more difficult to model then flat fading channels because each multipath signal must be modeled and the channel must be considered as a linear filter [2].

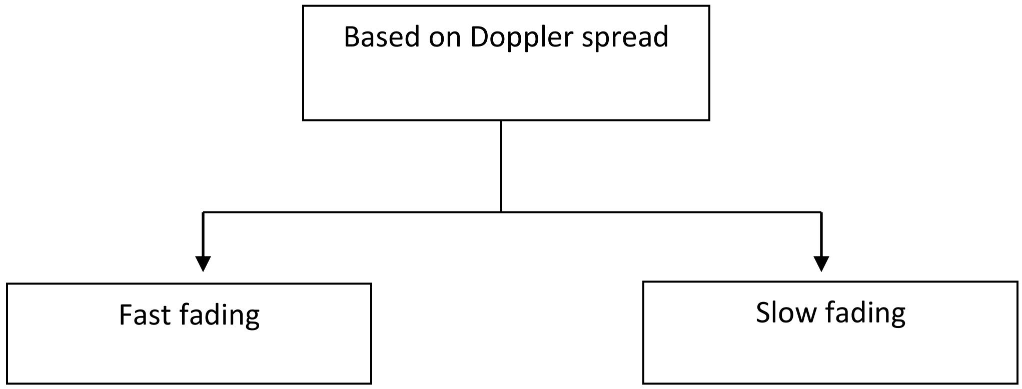

Figure-3: Fading Based on Doppler spread

Fast fading:

If the coherence time of a channel is smaller then the symbol period of the transmitted signal then the fading is known as the fast fading [2]. In fast fading there are rapid changes in the transmitted baseband signal. And because of frequency dispersion Doppler spreading leads to distortion of the baseband signal. In fast fading the channel variation is much faster than the baseband signal variations.

Slow Fading:

In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal [2].Slow fading is caused by the shadowing effect caused by different obstacles like buildings, mountains etc. In different small areas the average varies from one small area to the next in a random manner. In slow fading there is low Doppler spread and the channel variation is slower than the baseband signal variations.

2.2.3 Rayleigh and Rician Distributions:

Rayleigh Fading Distribution:

In mobile radio channels, the Rayleigh distributions is commonly used to describe the statistical time varying nature of the received envelop of a flat fading signal or the envelope of an individual multipath component. It is well known that the envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution. The Rayleigh distribution has a probability density function (pdf) given by the following equation [2]:

$$p(r)= \frac{r}{\sigma^2}\, e^{-(\frac{r^2}{2\sigma^2})} \quad ;(0\geq r\geq\infty)$$

$$0 \qquad\qquad ;(r<0)$$

Where, σ is the rms value of the received voltage signal before envelope detection and σ^2 is the time average power of the received signal before envelope detection.

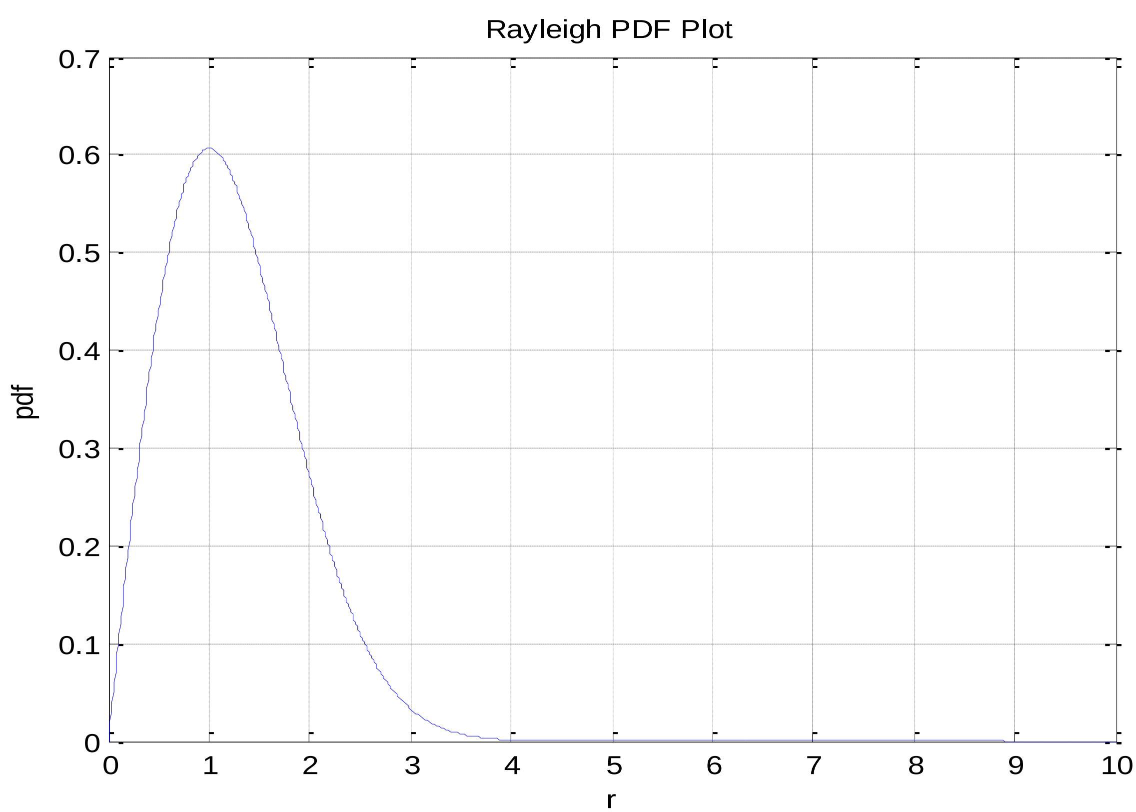

Figure-4: Rayleigh fading distribution

Rician Fading:

When there is a dominant stationary (none fading) signal component present, such as line-of-sight propagation path the small scale fading envelope distribution is Rician [2]. In such a situation random multipath component arriving at different angles are superimposed on a stationary dominant signal. At the output of an envelope detector, this has the effect of adding a dc component to the random multipath.

The Rician distribution is given by the following equation [2]:

$$p(r)= \frac{r}{\sigma^2} e^{-\frac{(r^2+A^2)}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) \quad ; \ (A \geq 0, r \geq 0)$$

$$0 \quad ;(r<0)$$

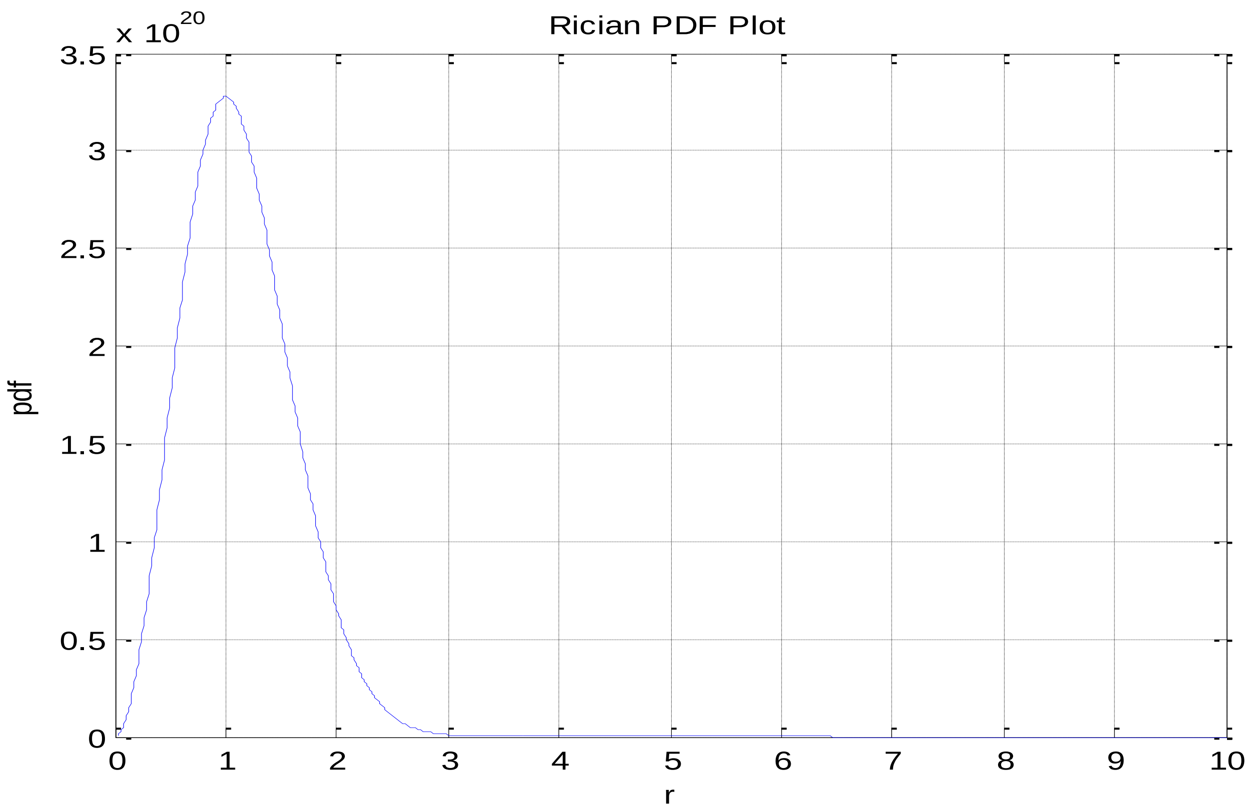

Figure-5: Rician fading distribution

CHAPTER 3

Degenerate forms of MIMO

The wireless communication system has gone through different generation starting from SISO (Single Input Single Output) systems to MIMO (Multiple Input Multiple Output) system. There are certain differences between SISO, SIMO, and MISO as transmission of high data rate is the main concern for a particular antenna. When the number of transmit and receive antenna increases, capacity is increasing. The major goal of wireless communication system is to create a strong network system with less capacity loss issue.

3.1 Single Input Single Output(SISO):

(i) This is the most simple antenna technology out of all four.
(ii) In SISO system one antenna is used at the source and similarly one antenna at the receiver.
(iii) There is no diversity and additional processing required.
(iv) The drawback of the system is it's performance. Fading and interference often impact the system lot more than a MIMO system.
(v) Application fields are: Wi-Fi, Bluetooth, TV, radio broadcasting.

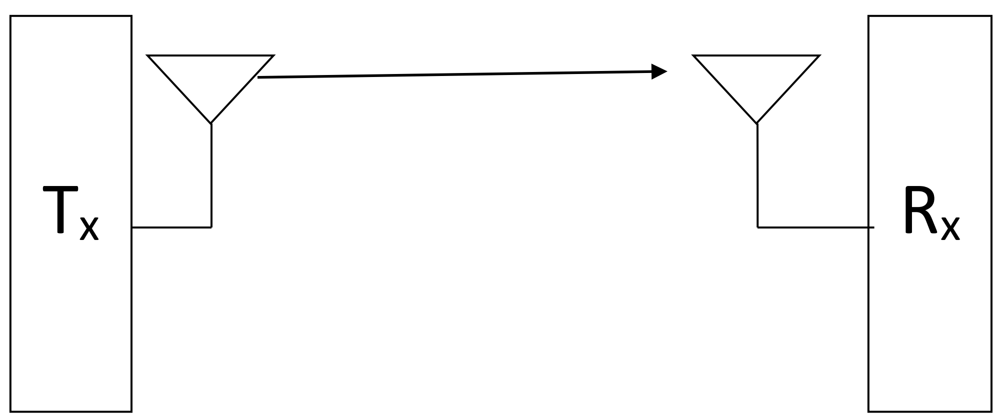

Figure-06: SISO Communication System

3.2 Single Input Multiple Output (SIMO):

This is also known as receiving diversity. In SIMO transmitter is equipped with single antenna and receiver has multiple antennas. SIMO is used in uplink transmission. SIMO has the advantage that it is relatively easy to implement although it does have some disadvantages in that the processing is required in the receiver. The SIMO systems are acceptable in many applications but where the receiving system is located in the mobile device like mobile phone, the performance me be limited by size, cost and battery drain[3]. SIMO systems were used for short waves listening. SIMO technology has widespread applications in Digital Television (DTV), Wireless Local Area Networks (WLANs), Metropolitan Area Networks (MANs), and Mobile Communications.

Two forms of SIMO which can be used:

(i) **Switched diversity SIMO**: Search for the strongest signal and switches to that antenna .

(ii) **Maximum ratio combining SIMO**: This type of SIMO takes both signals and sums up them to give them a combination. Like this way, the signals from both the antennas contribute to the overall signal.

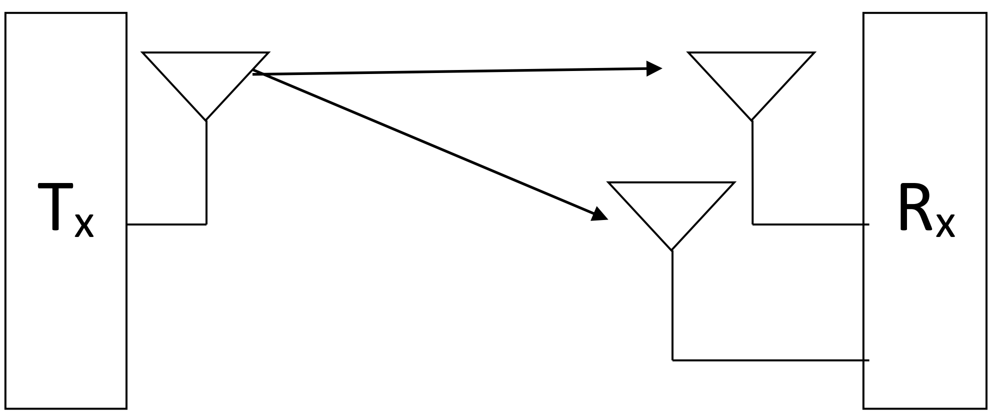

Figure-7: SIMO Communication System

3.3 Multiple Input Single Output (MISO):

MISO is one of the smartest antenna technology in which multiple antenna are mounted at transmitter with a single antenna at the receiving end. MISO systems has several advantage because the redundancy and coding has been shifted from receiving end towards the transmitting end and hence for examples of mobile phones, less power and processing is required at the user end or the receiver end[3]. These results less battery consumption as lower level of processing requires. MISO used in downlink transmission. The same data is transmitted redundantly from the two transmitter antennas. Hence, the receiver able to receive the optimum signal which it can then use to receive and extract the required data. When an electromagnetic wave interacts with cars, hills, high storage buildings and other obstacles, wave gets scattered and takes multiple paths to reach the destination. Such issues are known as multipath. This causes several issues like fading, losses and attenuation. Sometimes huge reduction in data speed and packet loss. The use of multiple antennas along with transmission of multiple signals in the transmitting side might reduce the trouble caused by multipath wave propagation.

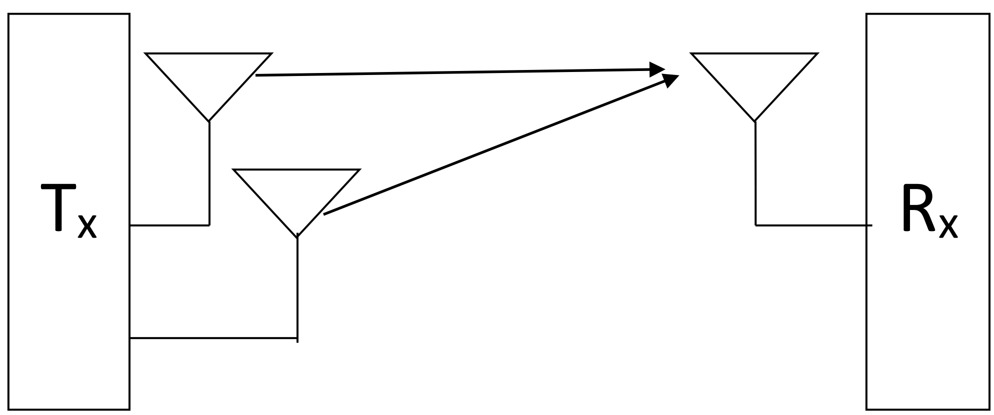

Figure-8: MISO Communication System

Chapter 4

Multiple Input Multiple Output (MIMO)

MIMO is one of the latest and important milestones in the development of wireless communication technology. MIMO leads to a significant increase in the data rate that is possible in the wireless communication system. Multiple in multiple out system is consist of multiple antenna both at the transmitting and receiving side where both the transmitting antenna working in the same frequency. MIMO technology has aroused interest because of it's possible applications in digital television, wireless local area networks (WLAN), metropolitan area networks (MAN) and mobile communications. MIMO specifically refers to a practical technique for sending and receiving more than one data signal with the same radio channel via multipath propagation simultaneously.

MIMO is effectively a radio antenna technology as it uses multiple antennas at the transmitter and receiver to enable a variety of signal paths to carry the data, choosing separate paths for each antenna to enable multiple signal paths to be used. Multiple transmit antennas can be used for beam forming and multiple receive antennas can be used for diversity. MIMO is an antenna system for wireless communications in which multiple antennas are used at both the sending and the receiving end. The antennas at each end of the communications circuit are combined to minimize errors and optimize data speed. A signal transmitted by a single MIMO antenna is received by all of the antennas of the receiving end and reflected signals are also received.

OFDM (Orthogonal Frequency Division Multiplexing): OFDM adds the orthogonal feature into multicarrier FDM. Orthogonality means do not cause interference with each other. In OFDM, the subcarrier is designed to be orthogonal. This allows subcarriers to overlap and saves bandwidth. Therefore, OFDM obtains both higher data rates and good spectrum efficiency.

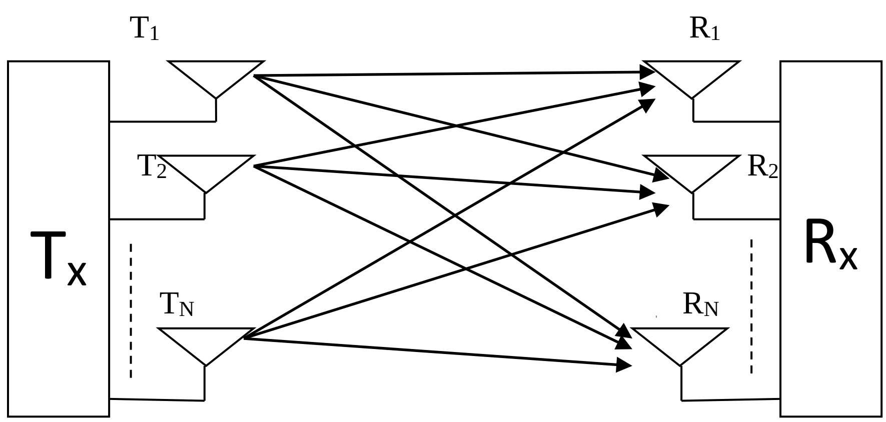

Figure-9: MIMO model

Considering, the MIMO model of Figure-9 with n number of transmitting and receiving antennas. That means nT transmit antennas and nR receive antennas. The channel matrix can be denoted by H complex matrix. The channel is determined by nT×nR matrix. The transmitted signal is represented by nT ×1 column matrix also denoted by symbol x and the received signal is represented by nR×1 column matrix denoted by symbol r.

Transmit vector x(n) is denoted,

$$\mathbf{x(n)} = [\, \vec{x}_1(n) + \vec{x}_2(n) + \ldots\ldots.. + \vec{x}_{nT}(n)]^T \tag{1}$$

The variance of the signal vector **x(n)** is represented by,

$$\sigma^2_x = \frac{\Sigma x_i^2(n)}{N_T}$$

$$= \frac{Total\ transmitted\ power}{N_T}$$

$$= \frac{P_T}{N_T}$$

$$\therefore\ \mathbf{P_T} = \sigma^2_x N_T \tag{2}$$

The received signal vector by the 1st antenna,

$$\mathbf{r_1(n)} = [\, \vec{x}_1(n)h_{11} + \vec{x}_2(n)h_{21} + \ldots\ldots.. + \vec{x}_{nT}(n)h_{n1}] \tag{3}$$

Complex channel matrix **H(n)** is represented by,

$$\mathbf{H(n)} = \begin{bmatrix} h_{11}(n) & h_{12}(n) & \ldots & h_{nT1}(n) \\ h_{21}(n) & h_{22}(n) & \ldots & h_{nT2}(n) \\ \vdots & \vdots & \ldots & \vdots \\ h_{nR1}(n) & h_{nR2}(n) & \ldots & h_{nTnR}(n) \end{bmatrix} \tag{4}$$

The noise vector n(n) is denoted,

$$\mathbf{n(n)} = [\vec{n}_1(n) + \vec{n}_2(n) + \ldots\ldots.. + \vec{n}_{nR}(n)]^T \quad (5)$$

The system can be represented as the matrix form,

$$\boldsymbol{r}(\boldsymbol{n}) = \boldsymbol{H}(\boldsymbol{n})\boldsymbol{x}(\boldsymbol{n}) + \boldsymbol{n}(\boldsymbol{n}) \quad (6)$$

To simplify the equation we can write,

$$\boldsymbol{r} = \boldsymbol{Hx} + \boldsymbol{n} \quad (7)$$

Where,
r=Receive vector,
H=Channel matrix,
x =Transmit vector,
and n= Noise vector

4.1 Signal -to-Noise Ratio (SNR):

Here, the correlation matrix is,

$$r_x = E[xx^H]$$

$$= \sigma^2_x I_{nT}$$

I_{nT} is the $N_T \times N_T$ identity matrix.

$$r_n = E[NN^H]$$

$$= \sigma^2_n I_{nR}$$

At the receiving end the Signal-to-Noise Ratio is,

$$SNR = \frac{Received\ signal\ power}{Noise\ power}$$

$$= \frac{|h|^2 P_T}{N_{Noise}}$$

$$= \frac{|h|^2 \sigma_x^2 N_T}{\sigma_n^2 N_R} \tag{8}$$

h(n) is the channel gain which is stationary.

4.2 Channel capacity:

The normalized channel capacity is expressed as,

$$C = \frac{1}{2} log_2(1 + SNR) \tag{9}$$

If the receiver has the knowledge of the channel state then channel capacity can be represented as,

$$C = E\left[\log_2\{1 + \frac{|h|^2 P_T}{\sigma_n^2}\}\right]$$

For MIMO,

$$C = E\{\log_2\left[\frac{\det(R_n + HR_xH^H}{\det(R_{n)}}\right]\}$$

MIMO channel capacity represented as,

$$C = \log_2\left\{\det\left(I_N + \frac{1}{\sigma^2} HR_SH\right)\right\} \text{bit/sec/Hz}$$

We can define a matrix,

$$Q = \begin{cases} HH^H & nR < nT \\ H^HH & nR \geq nT \end{cases} \tag{10}$$

Considering another two unitary matrices matrix U and V. Where the columns of **U** are the eigenvectors of $\mathbf{HH}^H$ and the columns of **V** are the eigenvectors of $\mathbf{H}^H\mathbf{H}$.

The received signal vector can be written as,

$$r = U\Lambda VHx + n; \tag{11}$$

Here, Λ is an $nR \times nT$ non-negative diagonal matrix, U and V are $nR \times nT$ and $nT \times nR$ unitary matrices respectively.

Λ can also be expressed as,

$$\Lambda = U^H H(VV^H)\, H^H U$$

If we define a new diagonal matrix,

$$\Lambda = [D\ 0]\,[D\ 0]^H$$

So, we can write,

$$r' = [D\ 0]\, x' + n \tag{12}$$

Where,

$$r' = U^H r$$

$$x' = V^H x$$

$$n' = U^H n$$

4.3 Alamouti Space-Time Block Coding:

It is a complex space-time diversity technique that can be used in MIMO mode. The Alamouti block code can achieve data rate of 1 and maximum diversity gain. Symbol sequence of two antenna elements is given below:

Time	**Transmitter 1**	**Transmitter 2**
t	$\mathbf{m_1}$	$\mathbf{m_2}$
t+T	$\mathbf{-m_2^*}$	$\mathbf{m_1^*}$

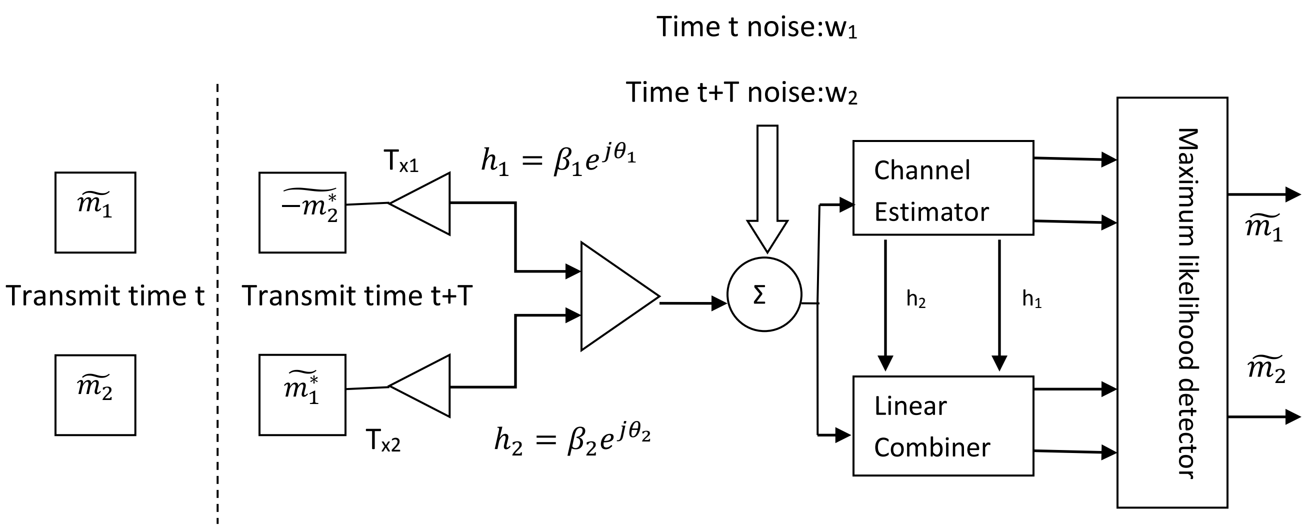

Figure-10: Block diagram of Alamouti Code

Here,

$h_1 = \beta_1 e^{j\theta_1}$

$\therefore h_1^* = \beta_1 e^{-j\theta_1}$

$h_2 = \beta_2 e^{j\theta_2}$

$\therefore h_2^* = \beta_2 e^{-j\theta_2}$

If, $\widetilde{m_1}$and $\widetilde{m_2}$ are transmitted at time t then,

$$\widetilde{x_1} = \beta_1 e^{j\theta_1}\widetilde{m_1} + \beta_2 e^{j\theta_2}\widetilde{m_2} + w_1 \tag{13}$$

If, $\widetilde{-m_2^*}$ and $\widetilde{m_1^*}$ are transmitted at time t+T then,

$$\widetilde{x_2} = -\beta_1 e^{j\theta_1}\widetilde{m_2^*} + \beta_2 e^{j\theta_2}\widetilde{m_1^*} + w_2 \tag{14}$$

$$\therefore \widetilde{x_2^*} = -\beta_1 e^{-j\theta_1}\widetilde{m_2} + \beta_2 e^{-j\theta_2}\widetilde{m_1} + w_2^* \tag{15}$$

$$\therefore \begin{bmatrix} \widetilde{x_1} \\ \widetilde{x_2^*} \end{bmatrix} = \begin{bmatrix} \beta_1 e^{j\theta_1} & \beta_2 e^{j\theta_2} \\ \beta_2 e^{-j\theta_2} & -\beta_1 e^{-j\theta_1} \end{bmatrix} \begin{bmatrix} \widetilde{m_1} \\ \widetilde{m_2} \end{bmatrix} + \begin{bmatrix} \widetilde{w_1} \\ \widetilde{w_2^*} \end{bmatrix} \tag{16}$$

From the equation,

$$X = H_\beta M + W \tag{17}$$

From the output of the linear combiner,

$$\begin{bmatrix} \widetilde{y_1} \\ \widetilde{y_2} \end{bmatrix} = \begin{bmatrix} \beta_1 e^{-j\theta_1} & \beta_2 e^{j\theta_2} \\ \beta_2 e^{-j\theta_2} & -\beta_1 e^{j\theta_1} \end{bmatrix} \begin{bmatrix} \widetilde{x_1} \\ \widetilde{x_2^*} \end{bmatrix}$$

$$= \begin{bmatrix} \beta_1 e^{-j\theta_1} & \beta_2 e^{j\theta_2} \\ \beta_2 e^{-j\theta_2} & -\beta_1 e^{j\theta_1} \end{bmatrix} \left(\begin{bmatrix} \beta_1 e^{j\theta_1} & \beta_2 e^{j\theta_2} \\ \beta_2 e^{-j\theta_2} & -\beta_1 e^{-j\theta_1} \end{bmatrix} \begin{bmatrix} \widetilde{m_1} \\ \widetilde{m_2} \end{bmatrix} + \begin{bmatrix} \widetilde{w_1} \\ \widetilde{w_2^*} \end{bmatrix} \right)$$

$$= \begin{bmatrix} \beta_1 e^{-j\theta_1} & \beta_2 e^{j\theta_2} \\ \beta_2 e^{-j\theta_2} & -\beta_1 e^{j\theta_1} \end{bmatrix} \begin{bmatrix} \beta_1 e^{j\theta_1} & \beta_2 e^{j\theta_2} \\ \beta_2 e^{-j\theta_2} & -\beta_1 e^{-j\theta_1} \end{bmatrix} \begin{bmatrix} \widetilde{m_1} \\ \widetilde{m_2} \end{bmatrix} + \begin{bmatrix} \beta_1 e^{-j\theta_1} & \beta_2 e^{j\theta_2} \\ \beta_2 e^{-j\theta_2} & -\beta_1 e^{j\theta_1} \end{bmatrix} \begin{bmatrix} \widetilde{w_1} \\ \widetilde{w_2^*} \end{bmatrix}$$

$$= \begin{bmatrix} \beta_1^2 e^0 + \beta_2^2 e^0 & \beta_1\beta_2 e^{-j\theta_1 + j\theta_2} - \beta_1\beta_2 e^{-j\theta_1 + j\theta_2} \\ \beta_1\beta_2 e^{-j\theta_2 + j\theta_1} - \beta_1\beta_2 e^{-j\theta_2 + j\theta_1} & \beta_1^2 e^0 + \beta_2^2 e^0 \end{bmatrix} \begin{bmatrix} \widetilde{m_1} \\ \widetilde{m_2} \end{bmatrix} + \begin{bmatrix} \widetilde{n_1} \\ \widetilde{n_2} \end{bmatrix}$$

$$= \begin{bmatrix} \beta_1^2 + \beta_2^2 & 0 \\ 0 & \beta_1^2 + \beta_2^2 \end{bmatrix} \begin{bmatrix} \widetilde{m_1} \\ \widetilde{m_2} \end{bmatrix} + \begin{bmatrix} \widetilde{n_1} \\ \widetilde{n_2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (\beta_1^2 + \beta_2^2) \begin{bmatrix} \widetilde{m_1} \\ \widetilde{m_2} \end{bmatrix} + \begin{bmatrix} \widetilde{n_1} \\ \widetilde{n_2} \end{bmatrix} \tag{18}$$

The reason Alamouti code is used to provide orthogonality. As from the above derivation we can clearly see that unwanted $\widetilde{m_2}$ is not present in $\widetilde{y_1}$. And like this also the unwanted $\widetilde{m_1}$ is not present in $\widetilde{y_2}$ and simplifies the receiver.

4.4 Water filling model:

The efficiency of the system can be enhanced with proper water filing model. The capacity of a MIMO system can further be increased by knowing the channel parameters both at the transmitter and at the receiver side and assign extra power at the transmitter by allocating the power according to the water filling algorithms to all the channels.

The method of water filling is like pouring the water in a bottle and each antenna of MIMO is considered as a bottle. And the dark portion is considered as noise and the white portion is the signal power.

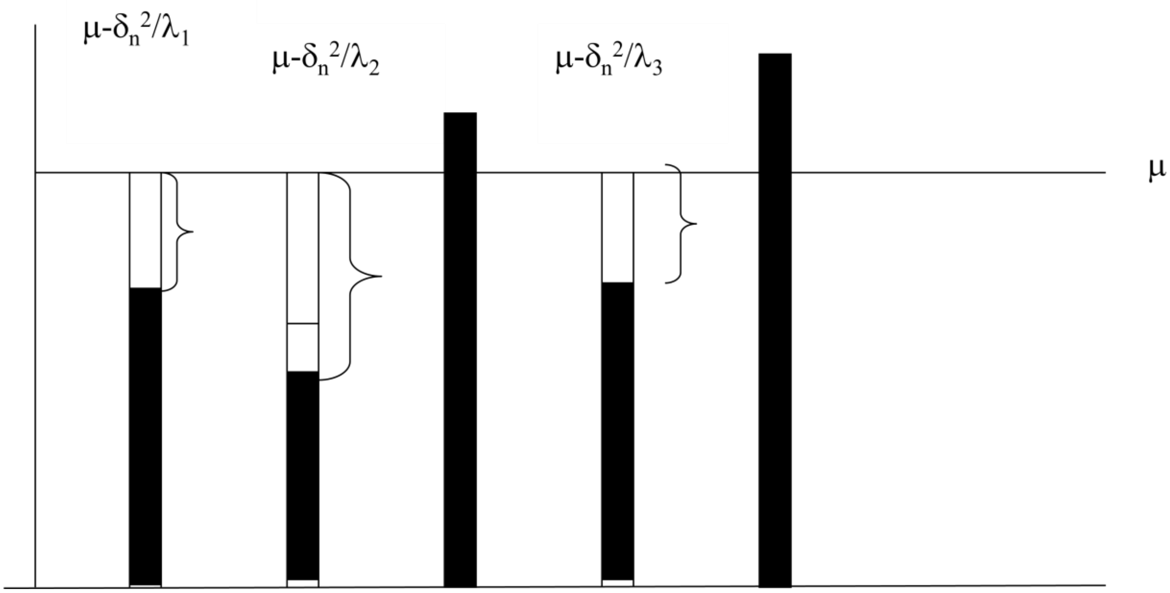

Figure-11: Waterfilling model

Here, on Figure-11 μ represents the power constraints. The total power of the bottle remains constant. Here, λ is the Eigen value and δ_n^2 is the variance of the noise.

Waterfilling is a metaphor for the solution of several optimization problems related to channel capacity. The simplest physical example is perhaps the case of spectral allocation for maximal total capacity under a total power constraint [4].

Power allocated by the individual channel is given by following equation derived in [5]:

$$\text{Power allocated} = \frac{P_T + \sum_{i=1}^{n} \frac{1}{H_i}}{\sum Channel} - \frac{1}{H_i} \tag{19}$$

Where P_T is the power of the MIMO system which is distributed among the different channels and H is the channel matrix of the systems. The Capacity of a MIMO system is algebraic sum of the capacities of all channels and is given by the following equation derived in [5]:

$$\text{Capacity} = \sum_{i=1}^{n} \log_2(1 + Powerallocated \times H) \tag{20}$$

We have to maximize the total number of bits to be transported, the results shows that the proposed water-filling power allocation scheme is. As per the scheme following steps are followed to carry out the proposed water filling algorithm.
Algorithm Steps:-

1. We do not need to reorder the MIMO-OFDM sub channel gain realization in a descending order
2. Take the inverse of the channel gains.
3. Water filling has non uniform step structure due to the inverse of the channel gain.
4. Initially take the sum of the Total Power Pt and the Inverse of the channel gain .It gives the complete area in the waterfilling and inverse power gain.

$$P_T + \sum_{i=1}^{n} \frac{1}{H_i} \tag{21}$$

5. Decide the initial water level by the formula given below by taking the average power allocated (average water Level)

$$\frac{P_T + \sum_{i=1}^{n} \frac{1}{H_i}}{\sum Channel} \tag{22}$$

6. The power values of each sub channel are calculated by subtracting the inverse channel gain of each channel.

$$\text{Power allocated} = \frac{P_T + \sum_{i=1}^{n} \frac{1}{H_i}}{\sum Channel} - \frac{1}{H_i}$$

7. In case the Power allocated value becomes negative stop the iteration process [6].

Chapter 5

Result

5.1 Without considering MIMO:

5.1.1 Rayleigh fading with different received signal:

We can plot the rayligh pdf in "MATLAB" by variying the received voltage signal σ .And also can see difference in the signals with the variation of received voltage signal σ. With increasing the received voltage signal σ the pdf decreases as fading takes place .

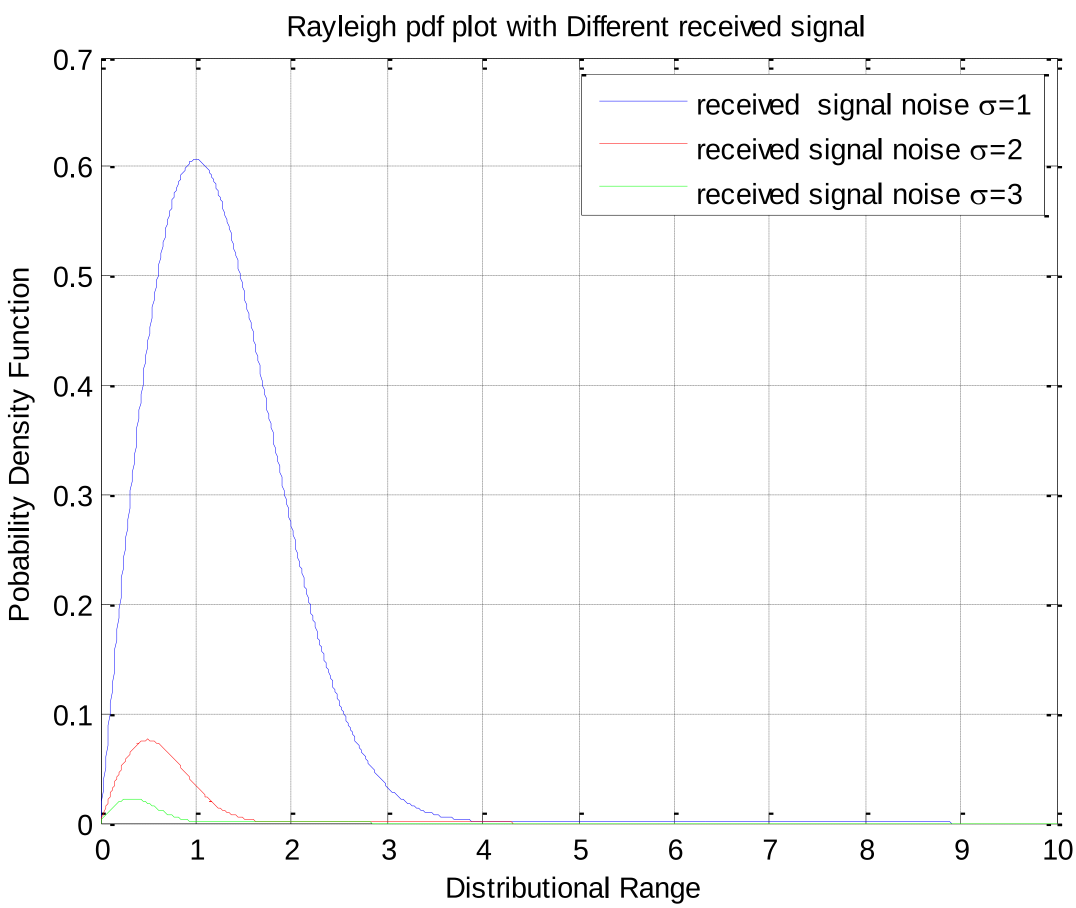

Figure-12: Rayleigh fading with different received signal

5.1.2 Rician fading with different received signal:

In rician fading pdf plot we can also plot the pdf by variying the received voltage signal σ. And also can see difference in the signals with the variation of received voltage signal σ. With increasing the received voltage signal σ the pdf decreases thus fading occurs.

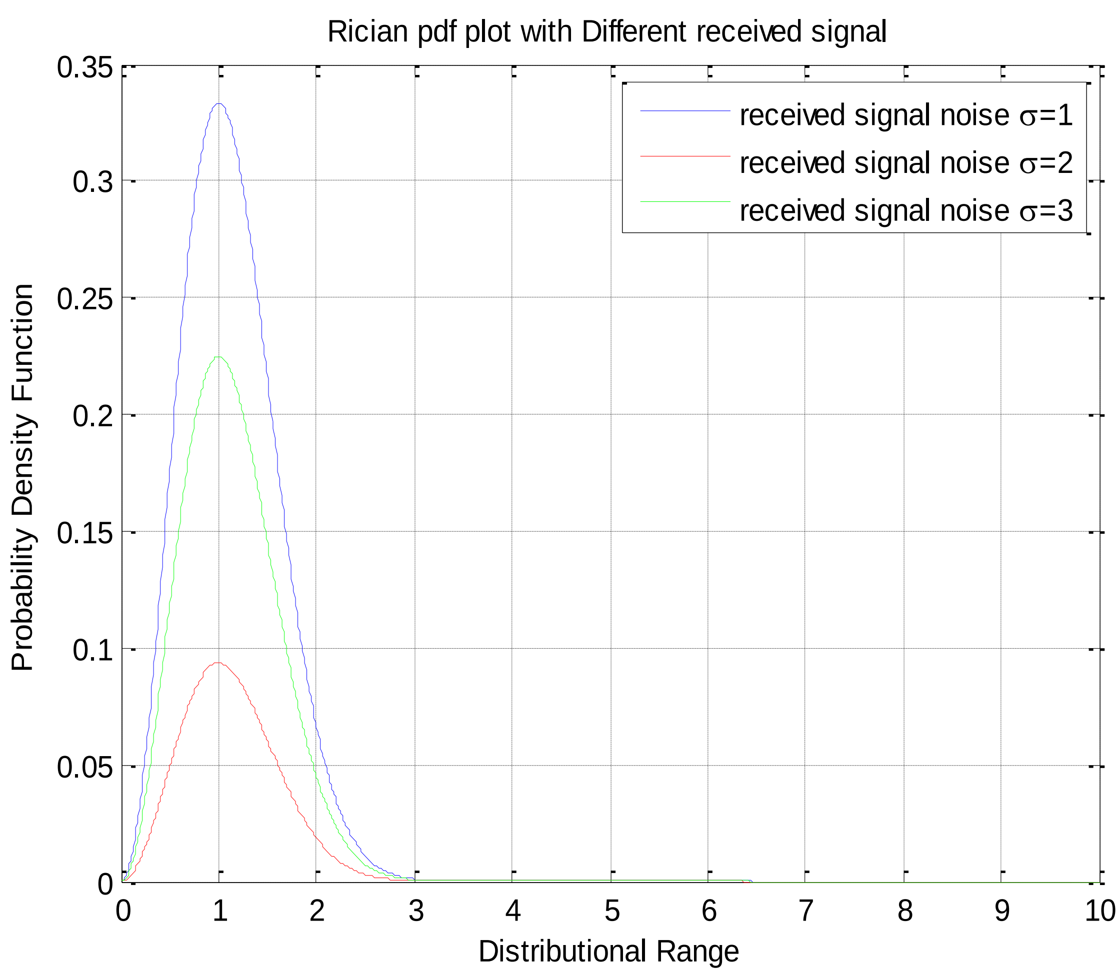

Figure-13: Rician fading with different received signal

5.2 Capacity allocation using MIMO:

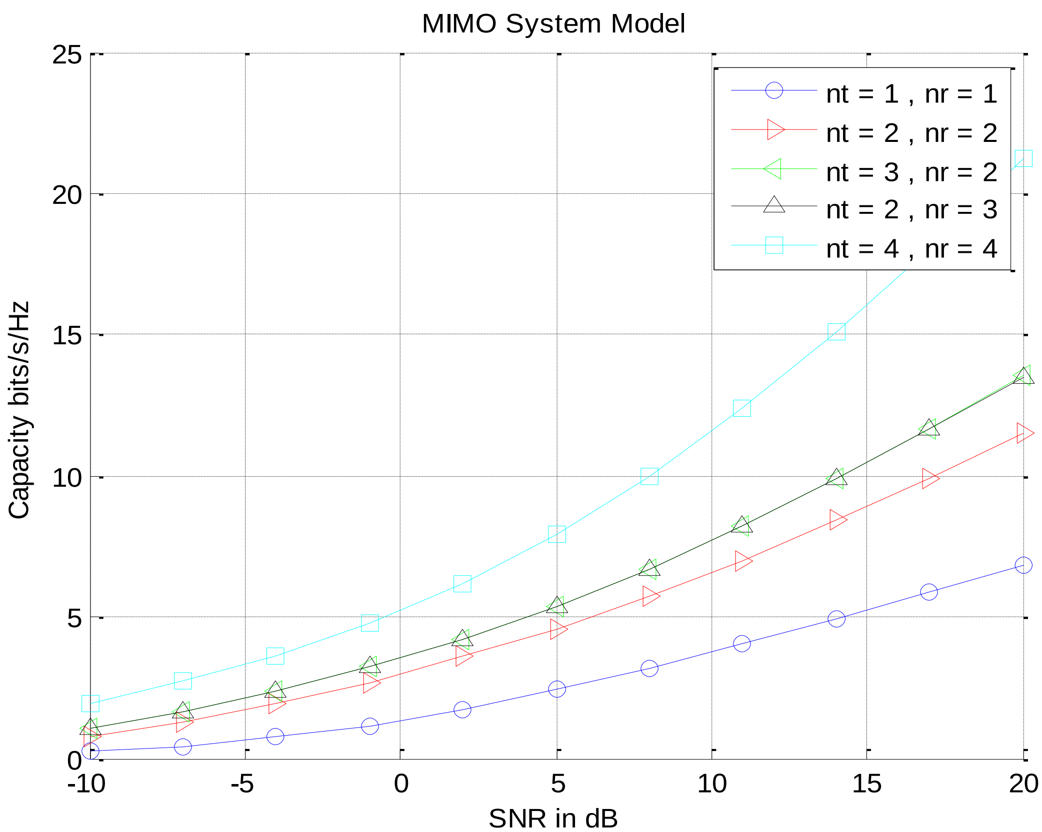

Figure-14: MIMO system model

In figure -14 a 4×4 MMO system is considered with 4 transmit and receive antennas. In the 1st curve (blue) we can see the capacity of the received signal of receiver antenna N_{R_1} which was transmitted from transmit antennaN_{T_1}. In the 2nd curve (red) we can see the capacity of the received signal of receiver antenna N_{R_2} which was transmitted from transmit antennaN_{T_2}. In the 4th curve (cyan) we can see the capacity of the received signal of receiver antenna N_{R_4} which was transmitted from transmit antennaN_{T_4}. But in the 3rd curve (green and black) we can see that if we send the same information using the transmit antennas N_{T_3} and N_{T_2} respectively and received by the received antennas N_{R_2} and N_{R_3} respectively we can see there is absolutely no change in the capacity of the signal in the curve with respect to SNR.

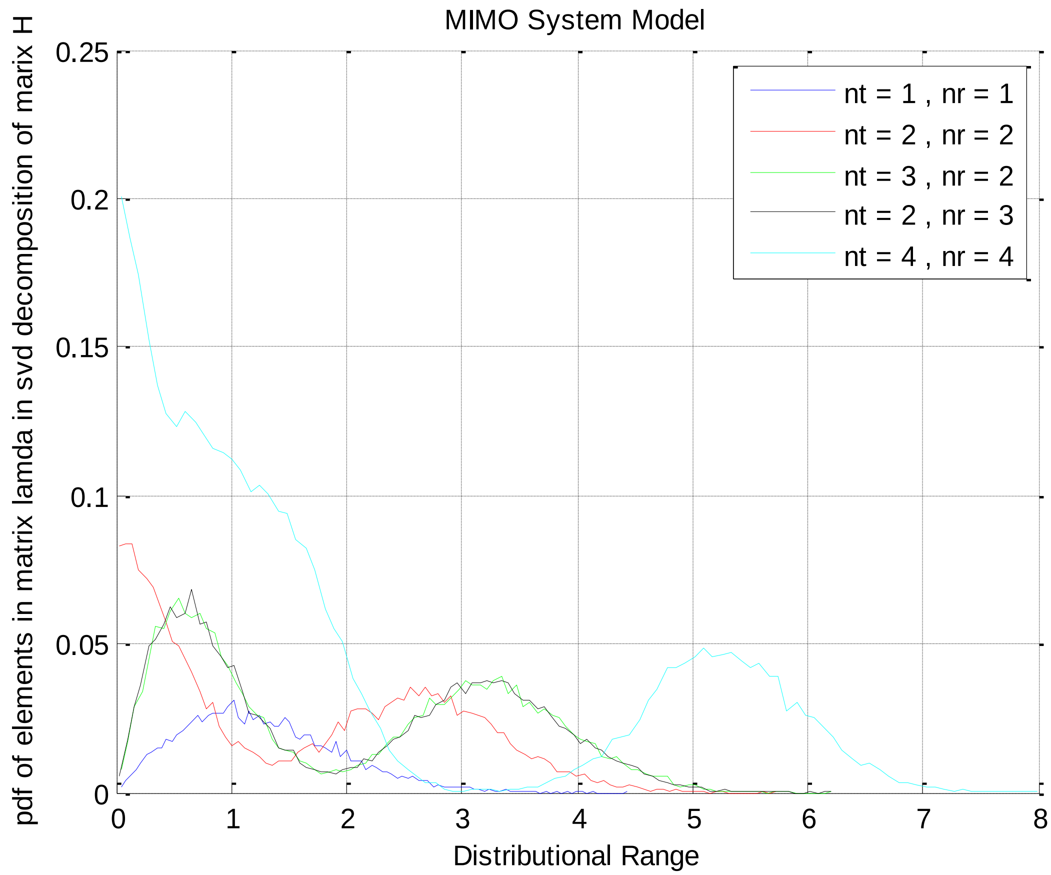

Figure-15: Waterfilling Model

Also in the figure -15 we can see the Probability Density Function (PDF) for the same transmit and receive antennas. And as we can see from the figure the probability density function for transmit antennas N_{T_3} and N_{T_2} respectively and received by the received antennas N_{R_2} and N_{R_3} are almost the same. Hence almost the same signal is received by the receive antennas. We can also see there is inter symbol interference between the signals and it will increase further with increase with the increasing number of transmit and receive antennas.

Chapter 6

Conclusion

In this research the performance analysis of MIMO system is studied using the waterfilling model. Here we considered a 4×4 MIMO system which provide us increased capacity and gives almost the same capacity like an antenna in line of sight (LOS) showed in figure-14 and figure-15. The capacity can be further increased by using more number of transmit and receive antennas. Capacity increases linearly with signal-to-noise ratio (SNR) at low SNR, but increases logarithmically with SNR at high SNR. Also it should be considered that if the number of transmit and receive antennas are increased then the inter symbol interference (ISI) will also increase. MIMO systems enable high spectral efficiency at much lower required energy per information bit. In future we can use MIMO in cognitive radio network, Long Term Evaluation (LTE) there is a future plane for LTE called massive MIMO systems, which employ a far larger number of antenna elements at the base station and eventually even more it will increase spectral efficiency, at least doubling, with 5 to 10 time.

Appendix

7.1 Raleigh fading pdf plot MATLAB code:

```
sig=1; %RMS value of received voltage signal
r = 0:0.01:10; %the range of value of 'r'
pdf=(r/sig.^2) .* exp(-r.^2/2*(sig.^2)); % the Rayleigh PDF
figure;
plot(r,pdf,'b') %Plotting the Rayleigh PDF
title('Rayleigh PDF Plot');
 grid on;
xlabel('r');
ylabel('pdf');
```

7.2 Rician fading pdf plot MATLAB code:

```
r=0:0.01:10;
v=5;
I0=0.55;
sig = 2; %the range of value of 'r'
pdf=(r./sig.^2) .* exp(-r.^2+v.^2./2*(sig.^2)).*I0.*(r.*v./(sig.^2)); % the rician PDF
figure;
plot(r,pdf,'b') %Plotting the rician PDF
title('Rician PDF Plot');
xlabel('r');
ylabel('pdf');
grid on;
```

7.3 Rayleigh fading with different received signal plot MATLAB code:

```
sig1=1; %RMS value of received voltage signal

sig2=2;

sig3=3;

r = 0:0.01:10; %the range of value of 'r'

pdf1=(r/sig1.^2) .* exp(-r.^2/2*(sig1.^2));

pdf2=(r/sig2.^2) .* exp(-r.^2/2*(sig2.^2));

pdf3=(r/sig3.^2) .* exp(-r.^2/2*(sig3.^2));

plot(r,pdf1,'b',r,pdf2,'r',r,pdf3,'g');

title('Rayleigh pdf plot with Different received signal');

legend('received voltage signal \sigma=1','received voltage signal \sigma=2','received voltage
signal \sigma=3');

xlabel('Distributional Range ');

ylabel('pdf');

grid on;
```

7.4 Rician fading with different received signal plot MATLAB code:

```
r=0:0.01:10;

v=1;

I0=0.55;

sig1=1; %the range of value of 'r'

sig2=2;

sig3=3;

pdf1=(r./sig1.^2) .* exp(-r.^2+v.^2./2*(sig1.^2)).*I0.*(r.*v./(sig1.^2));

pdf2=(r./sig2.^2) .* exp(-r.^2+v.^2./2*(sig2.^2)).*I0.*(r.*v./(sig2.^2));

pdf3=(r./sig3.^2) .* exp(-r.^2+v.^2./2*(sig3.^2)).*I0.*(r.*v./(sig3.^2));
```

```
title('Rayleigh pdf plot with Different received signal');

plot(r,pdf1,'b',r,pdf2,'r',r,pdf3,'g');

legend('received voltage signal \sigma=1','received voltage signal \sigma=2','received voltage signal \sigma=3');

xlabel('Distributional Range');

ylabel('pdf');

title('Rician pdf plot with Different received signal');

grid on;
```

7.5 Waterfilling algorithm MATLAB code:

```
function [Capacity PowerAllo] = WaterFilling_alg(PtotA,ChA,B,N0);
%
% WaterFilling in Optimising the Capacity
%===============
% Initialization
%===============
ChA = ChA + eps;
NA = length(ChA);     % the number of subchannels allocated to

H = ChA.^2/(B*N0);  % the parameter relate to SNR in subchannels
% assign the power to subchannel
PowerAllo = (PtotA + sum(1./H))/NA - 1./H;
while(length(find(PowerAllo < 0 ))>0)
   IndexN = find(PowerAllo <= 0 );
   IndexP = find(PowerAllo > 0);
   MP = length(IndexP);
   PowerAllo(IndexN) = 0;
   ChAT = ChA(IndexP);
   HT = ChAT.^2/(B*N0);
   PowerAlloT = (PtotA + sum(1./HT))/MP - 1./HT;
   PowerAllo(IndexP) = PowerAlloT;
end
PowerAllo = PowerAllo.';
Capacity  = sum(log2(1+ PowerAllo.' .* H));
title('MIMO Model')
```

7.6 MIMO system MATLAB code:

```
% in this programe a highly scattered enviroment is considered. The
% Capacity of a MIMO channel with nt transmit antenna and nr recieve
% antenna is analyzed. The power in parallel channel (after
% decomposition) is distributed as water-filling algorithm

% the pdf of the matrix lanada elements is depicted too.

clear all
close all
clc

nt_V = [1 2 3 2 4];
nr_V = [1 2 2 3 4];

N0 = 1e-4;
B  = 1;
Iteration = 1e4; % must be grater than 1e2

SNR_V_db = [-10:3:20];
SNR_V    = 10.^(SNR_V_db/10);

color = ['b';'r';'g';'k';'c'];
notation = ['-o';'->';'<-';'-^';'-s'];

for(k = 1 : 5)
    nt = nt_V(k);
    nr = nr_V(k);
    for(i = 1 : length(SNR_V))
        Pt = N0 * SNR_V(i);
        for(j = 1 : Iteration)
            H = random('rayleigh',1,nr,nt);
            [S V D] = svd(H);
            landas(:,j)  = diag(V);
            [Capacity(i,j) PowerAllo] = WaterFilling_alg(Pt,landas(:,j),B,N0);
        end
    end

    f1 = figure(1);
    hold on
    plot(SNR_V_db,mean(Capacity'),notation(k,:),'color',color(k,:))

    f2 = figure(2);
    hold on
```

```
    [y,x] = hist(reshape(landas,[1,min(nt,nr)*Iteration]),100);
    plot(x,y/Iteration,'color',color(k,:));
    clear landas
end

f1 = figure(1)

legend_str = [];
for( i = 1 : length(nt_V))
    legend_str =[ legend_str ;...
        {['nt = ',num2str(nt_V(i)),' , nr = ',num2str(nr_V(i))]}];
end
legend(legend_str)
grid on
set(f1,'color',[1 1 1])
xlabel('SNR in dB')
ylabel('Capacity bits/s/Hz')

f2 = figure(2)
legend(legend_str)
grid on
set(f2,'color',[1 1 1])
ylabel('pdf of elements in matrix lamda in svd decomposition of marix H')
xlabel('Distributional Range')
title(‘Waterfilling Model’)
```

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