A Markov-Switching model of Taka/Rupee exchange rate: estimation and forecasting

Raisa Shafiquddin

Thesis Supervisor

Dr. Syed Abul Basher

A thesis presented for the degree of Masters of Social Science

Department of Economics
East West University
Bangladesh

2016

Acknowledgement

I am thankful to almighty Allah for being merciful and gracious towards my endeavors. I am eternally thankful to my loving parents and all my teachers, here at East West University and elsewhere, who have been sources of immense inspiration and support all throughout my academic journey. A special thanks to my thesis supervisor, Dr. Syed Abul Basher for giving me the opportunity to explore the complex and intricate world of exchange rate economics. This would not have been possible without his guidance and thorough supervision. Without further ado, here I begin.

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Date: ______________

Signature: ______________

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Name: Raisa Shafiquddin
Degree: Master of Social Sciences in Economics
Title: A Markov switching model of Taka/Rupee exchange rate: estimation and forecasting

Examining Committee:

____

Dr. Syed Abul Basher
Thesis Supervisor
Chairperson, Department of Economics

____

Dr. A.K. Enamul Haque
Professor, Department of Economics

Date Approved: ______________________

Thesis title: *A Markov switching model of taka/rupee exchange rate: estimation and forecasting*

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Table of Contents

List of tables

List of figures

Modeling and forecasting Taka/Rupee exchange rate using monetary variables and trade balance

A Markov switching approach

Abstract

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1. Introduction

The study of exchange rate in international economics is a widely contested topic. An exchange rate being the relative amount of one currency with respect to another can have diverse impact on an economy and more so with rise of trade and transactions between nations (Nicita, 2013). Simply put exchange rate functionality in an economy is like just another price variable that is adjusted in the market through virtues of demand and supply and is subject to market shocks being traded almost 24 hours a day at some end of the world. Given this fluctuating nature, exchange rates are by general consensus hard to predict and therefore demonstrates little connection with its fundamentals such as price differential, inflation and interest differential. In other words, that would mean exchange rate today is the best guess for tomorrow's rate or it has the random walk characteristic.

If the statement above is indeed true then there might not be much use of economics or economic models to forecast exchange rate. In fact, the difficulties in international macroeconomics to forecast exchange rate with structural models has been documented as early as the 80's and is still an ongoing debate in this field. Meese and Rogoff (1983) was the first to refute monetary exchange rate model and other monetary variables such as PPP and UIP on the ground that it fails to predict the future path of exchange rate. The disconnect puzzle or the Meese and Rogoff puzzle as commonly known in exchange rate literature serves as an empirical evidence of poor out-of-sample forecasting performance of aforementioned linear exchange rate models. Several other authors followed suit with similar empirical evidence (Chin and Meese (1995); Meese and Rogoff (1988)). Regardless, the academic world deemed it too soon to draw such conclusion about the validity of these models and the literature thus embarked on a long journey to search a proper specification that would increase the predictive ability of the existing theoretical models. Over the last two decades, there are some papers that emphasized on the role of expectation on exchange rate variability while others focused on the importance of selecting better predictors (e.g. Gournichas and Rey, 2007; Molodtsova and Papell, 2009) and yet some others advocated the use of new test procedures (e.g. Clark and West, 2007).

Specific to monetary exchange rate model, analysis and debate ensue surrounding both short run and long-run movements of the model. With the long-horizon predictive ability of the monetary exchange rate model to some extent being established with the works of Mark (1995), Mac Donald and Taylor (1994) in the last decade, short-run horizon, that is more significant in terms of policy implication, has become the subject of exploration amongst researchers in recent times. Can fundamentals explain and predict exchange rate in the short-run? To investigate this particular question, the application of nonlinear time series models has been useful. Frommel et. al (2005) particularly applies Markov switching approach to conduct nonlinear modeling of fundamentals on US dollar exchange rates and find monetary fundamentals to have sufficient amount of explanatory power in comparison to a linear model. The authors in this study are driven by one key factor and that is, fundamentals are time varying parameters and thus once modeled in a regime switching process, it can explain exchange rate movements better as opposed to the constant coefficient estimates of linear models. Engel and Hamilton (1989) was the first to apply MS model and invent the so called "stochastic segmented trends" in exchange rate data.

This study is motivated by presence of nonlinearities in exchange rate data and the recent success of Markov switch model in the context of exchange rate modeling. Empirically, the study employs the Markov switching framework and investigates the impact of a set of monetary fundamentals such as interest rate differential, inflation differential, money growth differential on taka/rupee exchange rate during the period of May 2003-March 2016. The nominal exchange rate of taka/rupee interests us as India is a key trading partner for Bangladesh. Another interesting addition in our study is that we modify the standard monetary exchange rate model by augmenting a trade balance variable in it to analyze its impact on exchange rate estimation and prediction. The main motivation of using this variable in our model is the widening discrepancy in trade figures between India and Bangladesh. In the recent decade, India has emerged as the second most important destination of import for Bangladesh but Bangladesh's export to India is as low as 0.1% in the global import of India. Even though exports have shown some growth in recent times, the bilateral trade deficit continues to soar (between FY2004-05 and FY2012-13, trade deficit has doubled from US$1882 million to US$ 4176 million)[1]. The second issue of the study is to analyze if Markov switch model improves forecastability of the traditional model against the benchmark of random walk specification. In this regard, we test the null hypothesis of equal predictive ability between the competing models by conducting a few out-of-sample forecasting tests via different forecasting windows such as static, dynamic, rolling and recursive. To gauge forecast accuracy, we apply the statistical measures of predictability such as mean square error (MSE) and mean average error (MAE) and assess the statistical significance of the out-of-sample MSE/MAE of the forecasts generated by the models using Diebold and Mariano (1995) test statistic.

By applying 2-state Markov switching framework to monetary exchange rate model, we find evidence of nonlinear relationship between monetary fundamentals and exchange rate as demonstrated by the highly persistent appreciation and depreciation regimes. In particular, the monetary fundamentals explain the behavior of Taka/Rupee exchange rate well as the interpretation of coefficients render statistical significance for inflation differential, interest differential, money growth differential and also for the non-monetary fundamental of trade balance. In fact, the trade balance variable improves the fit of our model according to the regime classification measure (RCM) values. The fundamentals used in the study are also in line with the theory except for money growth differential, the coefficient of which shows a negative relationship with exchange rate movement. Empirical evidence from the out-of-sample forecasting exercise provides a mixed verdict. According to Diebold Mariano (DM) test statistic, MS monetary model outperforms random walk for the one-month ahead forecast errors

[1] The stylized facts on Bangladesh-India bilateral trade has been gathered from Rahman and Akhter (2016)

generated using a rolling window indicating predictive content of monetary fundamentals.. This positive result, however, is not supported by the recursive window, the DM results for which are statistically significant for the random walk model. On the other hand, static and dynamic window mostly delivers insignificant MSE and MAE differences.

The study is organized as follows. In section 2, we discuss exchange rate modeling and forecasting over the past 25 years with a focus on exchange rate series from a developing country perspective. Section 3, 4 and 5 discusses the theory, data and methodology employed in this study. In section 6, we estimate the Engel and Hamilton's Markov switch model with monetary fundamentals as exogenous variables and compare its forecast accuracy with the benchmark model of random walk. Section 7 draws the conclusion.

2. Literature Review

2.1 Structural models of exchange rate determination

One of the earliest ways of exchange rate determination was the flow approach-the traditional view that focused on demand and supply of trade flows in foreign exchange rate market. However, with Bretton Woods summit's decision to depart from fixed exchange rate system to floating exchange rate, the flow approach soon lost its validity in the theoretical world and the asset based models of the 1970's such as PPP, UIRP, and monetary model became the more predominant view to exchange rate determination. In addition, arrived different variants of the monetary models such as the class of portfolio balance models[2] of Hooper-Morton (1982) and Frankel (1985), real differential model that each had monetary approach as a special case. These structural models are based on the common, underlying assumption of rational expectation and perfect capital mobility.

Modeling exchange rate with structural elements came under scrutiny with the advent of the disconnect puzzle by Meese and Rogoff. Meese and Rogoff (1982) demonstrates how the traditional asset based models fail to provide out of sample forecasts by root mean square error(RMSE) criteria[3] which lead the authors to conclude that there is considerable amount of disconnect between exchange rate and macroeconomic fundamentals. The literature following Meese and Rogoff is divided in opinion and progressed gradually through various attempts. As Boughton (1988) specifically argues some of the empirical problems associated with the monetary approach may be solvable by paying more attention to the specification of the empirical relationships without calling into question the underlying monetary theory.

If it's theory that must not be questioned, then much of the weight of the ongoing debate surrounding structural model's predictive ability shifts to methodological improvements. We present some arguments of the debate in this section, not always in chronological order. For example, Cheung, Chinn and Pascual (2005) tests the out-of-sample efficacy of some of the

[2] These are the non-monetary class of asset-based models that assumes imperfect substitutability between domestic and foreign bonds by risk-averse agents

[3] RMSE is a statistical criteria of measuring the difference between sample values and predicted values typically used to evaluate forecast accuracy

models of the nineties against benchmark model of random walk together with sticky price and purchasing power parity using both error correction and first difference specification and finds evidence of long-horizon predictability of exchange rate movement but not short-run. Similarly, Mark (1995) uses Gaussian parametric and non-parametric estimate of the simple monetary exchange rate model and finds random walk characterization at 1- and 4-quarter horizon but long horizon predictability at 16 quarter. However, Killian (1999) modifies the bootstrap method employed by Mark in a vector error correction framework and argues that because of presence of nonlinearities in the data generating process, the bootstrap p-values of Mark's long horizon regression results are biased and thus mistakenly advocate long-horizon predictability of the monetary model.

Mark and Sul (2001) implements panel specification in monetary model to mitigate previous confounding results and examines if predictability of the model improves once cross-country shocks are accounted for. The authors apply the specification on a panel of nineteen countries with inferences being drawn from both asymptotic and bootstrap distribution and find that exchange rates are co-integrated and with regard to forecastibility of the specification there seems to be sufficient predictive power of monetary fundamentals in an out-of-sample experiment generated from panel regression. Basher and Westerlund (2006) corroborate the work of Mark and Sul (2001) and puts emphasis on the inference end of the test statistics employed to panel data sets in the literature. By pooling parameters of both the forecasting equation and the test statistics, the authors come to the conclusion of a larger power gain and hence better exchange rate predictability of the monetary model than previous studies.

Besides monetary model and panel specification, other structural models that have seen modest success at improving exchange rate forecast are external balance model by Gournichas and Rey (2007) and Taylor rule fundamentals of Molodtsova and Papell (2008). Gournichas and Rey (2007) uses ratio of net exports to net foreign assets as a trade balance variable to forecast one-period-ahead forecasts of both trade and FDI-weighted exchange rate. The results of the study render statistically significant test inferences of bootstrapped CW, DMW and ENC-NEW test which are also robust to varying forecast window as reported in Rogoff and Stavrakeva (2008). Molodtsova and Papell (2008) tests out-of-sample predictability of OECD countries currencies (USD as the numeraire) in a Taylor rule specification with inflation gap, output gap and interest rate as right hand side variables. The study provides evidence in favor of Taylor rule model that yields higher forecast accuracy than random walk specification when inferences are made with Clark-West procedure.

2.2 Non-linear modeling of exchange rate and fundamentals

Figure1. Nonlinearities in exchange rate data

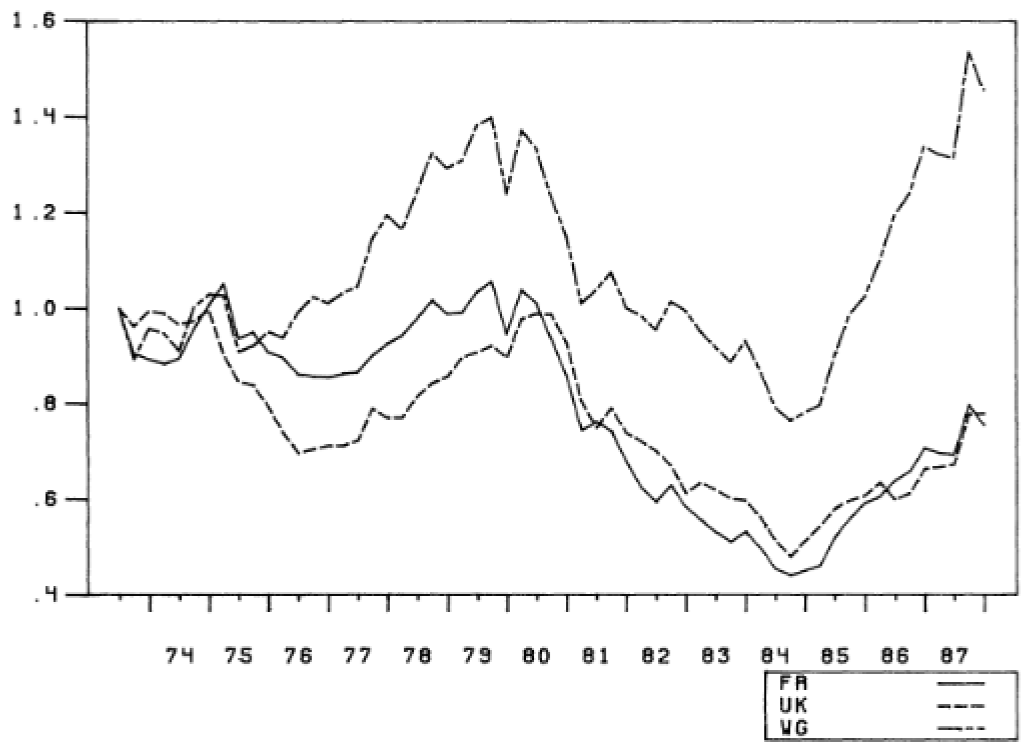

Figure 1. Stochastic segmented trends in dollar/mark, dollar/pound and dollar/franc exchange rates. Adapted from "Long swings in the dollar: are they in the data and do markets know it" by C. Engel and J. D. Hamilton, 1990, *American Economic Review, 80*, p. 690. Copyright 1990 by the American Economic Review.

Since late 80's and early 90's yet another body of academic literature emerged that aimed on exploiting non-linearities[4] in exchange rate process through Markov switching models- a branch of statistical model that could capture "swings" or "segmented trends", a typical characteristic demonstrated by exchange rates after regime shift to floating exchange rate system. It must be taken into account that the presence of non linearity in exchange rate data was first detected by Hseih (1989)[5]. In the purview of non-linear models and its application on exchange rate data, the aforementioned concept of long swings was first formalized by Engel

[4] Non linearity in time series is a feature. It should be noted that there are distinct types of nonlinearity. According to Kaufmann et al. (2014), Markov switching dynamics are better for capturing "sudden" but "persistent" shocks as in developing countries and models such as ESTAR for large deviation from PPP as noticed in developed countries

[5] Hseih (1989) employed Brock, Dechert and Scheinkman (BDS) test to establish nonlinear dependence in daily foreign exchange rates and thereby made comparison between types of nonlinearity.

and Hamilton (1990) with the particular application of Markov switch model on US dollar exchange rate as depicted in Figure 1 above.

Amongst subsequent studies employing Markov Switch model, there exists some empirical evidence in favor of nonlinear relationship between exchange rate and fundamentals albeit the body of work to some extent is limited. In this regard, Cushman (2000) is an exception. Beckmann and Czudaj (2014) methodologically improves Cushman's results and considers a multivariate framework for monetary exchange rate model for Canada/US exchange rate within a MS-VECM[6] framework to analyze how exchange rate adjusts to fundamental deviations and the authors find empirical evidence of long-run relationship between exchange rate and fundamentals. Frömmel, MacDonald, and Menkhoff (2005) studies Markov switching regime in a monetary exchange rate model to explore which factors drive regime switches using Frankel's 1979 variant of the model(MS-RID). The result shows evidence of nonlinear relationship between monetary fundamentals and USD exchange rate of three currencies: Mark, Yen and Pound Sterling. The time-varying coefficients of MS-RID are statistically significant while the constant coefficients of RID model are not. More recently, Wu (2015) further corroborates the modeling aspect of the studies cited above. While analyzing Asia Pacific country currencies he incorporates time-varying transitional probabilities (TVTP)[7] in the Markov switch model and finds higher log-likelihood[8] values for the model compared to a MS-RID. There is also evidence of statistically significant variables in one of the regimes reconfirming economic fundamentals are time varying. Grauwe and Vanteenkiste (2001) draws a distinction of the relationship between exchange rate and fundamentals such as money supply, inflation and interest rate by using monthly and quarterly data of both high and low inflation countries in a Markov switch-autoregressive(MS-AR) framework. The results generate a certain pattern- high inflation countries are marked by less frequent regime change while low inflation countries demonstrate frequent structure break which lead the author to come to the conclusion that structural model might work better for high inflation countries than low inflation countries.

Exchange rate forecasting has also sparked diverse results and opinions in the literature. The predictability depends on multiple factors such as selection of key predictors(e.g. better predictors for "appreciating" and "depreciating" regime), forecast horizon-one step or multi-

[6] A vector error correction model augmented with Markov switching in mean and variance

[7] An extension of the Markov switching model which allows the transition probabilities to vary subject to certain lagged observation

[8] Log-likelihood , a model selection criteria, that basically stands for log of the likelihood, that is, parameter estimates of a model that increase the occurrence of the data. Closer the log-likelihood value is to 1, better the fit of the model.

step, sample period, data frequency, forecast evaluation method and the methodology employed (Rossi, 2013). Now, how effective is Markov switch, within a structural model framework, in forecasting exchange rate? The answer is somewhat ambiguous. The literature is filled with mixed evidence with some studies delivering positive forecasting performance while others nodding in disagreement. Engel (1990)'s univariate process of exchange rate regime has been well received in the exchange rate forecasting literature being the first of its kind in this field to establish the nonlinear relationship between dollar/mark, dollar/exchange rate and its past observation. However, it must also be noted that the study does not probe into the source of the switch and it also received criticism in subsequent literature on its capacity to forecast. For example, Engel (1994) (wrong prediction- it shows depreciation of USD during early 80s when it actually appreciated). Kaminsky (1993) emphasizes on the role of expectation and argues that investors in the foreign exchange market are informed, so, other than taking into account past observations of spot rate in the regime switching process as in Engel(1990), one must also focus into announcements made by monetary policy authorities to better predict exchange rate path. Markov switch/regime-switching process needs to incorporate such variables to predict better and accurately. Kirikos (2011) compares forecasting ability of linear and nonlinear model and finds random walk specification at short-horizon and linear structural model at long-horizon to be more apt candidate of forecasting exchange rate. Dacoo and Satchell(1999) applies the basic segmented trends model on DM/dollar exchange rate and analyzes as to why regime switching model have not been able to provide satisfactory results in the previous literature. The authors present an analytical discussion and argue that regime switching models can very easily have higher MSE than RW model due to even slight regime misclassification error.

On a different note, Chen and Lee (2006) justify the use of Markov switch model to predict exchange rate. By deriving a rational expectation model of exchange rate determination, the authors show that exchange rate process is a state-dependent phenomenon, the states being central bank's intervention and central bank's non-intervention.. In light of more recent literature, Nikolsko and Prodan (2014) extends the study of Engel(1994) over a larger data set of currencies of 12 OECD countries versus the dollar and reanalyzes forecastability of MS-RW model using alternative test statistics/new test procedure and finds evidence of both short-horizon and long-horizon predictability of the pure statistical model.

With regard to Markov switch testing the forecasting performance of monetary model, the study of Frommel et. al (2005) is relevant again. The authors compare the forecasting ability of three models using RMSE and MAE statistical measures, the three specifications being pure Markov switch model, Markov switch model with fundamentals and the benchmark of random walk and finds Markov switch with fundamentals with the best forecasting performance. At the same time, the authors forecast over multi-periods of 1, 3 and 6 months and find over short-run, random walk is the better model than Markov switch with fundamentals.

2.3 Developing and emerging country's exchange rate series modeling and forecasting: application of Markov Switch

In the previous subsections, we mostly discuss how exchange rate series have been modeled and forecasted with respect to developed country currencies. In this section, we take upon the issue keeping a focus on developing country currency analysis that has used the Markov switching framework. For example, Chen (2006) use currencies from six developing countries such as Indonesia, South Korea, Philippines, Thailand, Mexico and Turkey to establish the relationship between interest rate and exchange rate volatility by segmenting the data into high volatility and low volatility regime. The author finds that on increasing the interest rate, exchange rate volatility exhibits a tendency to shift to high volatility regime and thus the authors conclude that higher interest rate is not enough to safeguard exchange rate of these countries from a "crisis" phase. Sinha and Kohli(2013) in their study tries to establish relationship between India's foreign exchange rate market and stock market on one hand and on the other tries to look at how certain macroeconomic variables such as inflation differential, interest rate, current account deficit. . Bakin, Anwer and Khan (2013) carries out a forecasting exercise on daily Bangladeshi exchange rate series using various nonlinear models such as adaptive neuro fuzzy inference system (ANFIS), MS-AR and and GARCH model. The study compares forecasting performance of the aforementioned models through popular statistical measures such as MAPE and RMSE and concludes that ANFIS model possesses the highest forecast accuracy out of all three models. Shen and Chen (2006) apply Markov switching model on Taiwanese exchange rate and draw a distinction between the segmented trends of developed and developing country currencies- while developed economies have greater tendency to adhere to appreciation and depreciation regime, developing countries demonstrate persistence in the appreciation regime.

3. The theory

In this section, we begin by discussing two parity or no-arbitrage conditions of exchange rate determination in the goods and capital market, namely interest rate parity (IRP) and purchasing power parity (PPP). Next, a theoretical dissection of the monetary exchange rate model is undertaken-flexible, sticky price models are discussed with a focus on both the theoretical interpretation and empirical advancement over time.

3.1 Interest rate parity (IRP)

The economic theory of interest rate parity-also known as the International Fisher Effect as posited in Fisher (1930)-basically relates the percentage change in the exchange rate to the interest rate differential between two countries. Based on the joint hypothesis of risk neutrality and rational expectation on behalf of agents, the theory states that the differential between interest rates of two countries must be reflected in the differential between the spot exchange rate of those two countries so that there is no room for arbitrage option in the foreign exchange market .Let, $e_t^{Tk/Rupee}$ denote logarithm of the nominal exchange rate-domestic price of foreign currency[9]; E denotes percentage change in exchange rate; i_t and i_t^* denote the nominal interest rate on the same asset with h periods to maturity. Therefore, the IRP condition, in the context of this study, would be:

$$(e_t^{Tk/Rupee} - e_{t+1}^{Tk/Rupee}) = (i_t - i_t^*)$$

$$\text{OR,} \quad E = (i_t - i_t^*)$$

For IRP to sustain, two assumptions must be satisfied which are as follows:

- There must be easy capital mobility between countries
- The two assets in question must be complete substitutes of one another. For instance, the theory assumes that a deposit rate in a foreign bank is the same as a deposit rate in a domestic bank

Based on the above assumptions, in case of a difference between nominal returns/interest rate between foreign and domestic deposit, let's say nominal return at home is higher than that of the foreign counterpart, market investors would have the incentive to move their money to the

[9] An increase in the exchange rate would therefore mean depreciation of the home currency

bank that pays higher nominal return. IRP, therefore, only exists when the expected nominal rates are the same for domestic and foreign assets and hence, this parity condition is also otherwise known as no-arbitrage condition. In the event there is any difference between the nominal interest rates, the theory expects that an adjustment must occur through expected appreciation or depreciation in the foreign or domestic currency. For instance, if domestic interest rate is 5% and foreign interest rate is 3%, and then according to the theory, the investors expect foreign currency to *appreciate* by 2% or by the same count, investors expect the domestic currency to *depreciate* by 2%. Now to justify as to why domestic currency depreciate of the country with the higher interest rate, let us consider the aggregate money demand model. Money demand comes in to the picture as interest rate influences both individual and aggregate money demand. According to aggregate money demand model, a higher interest rate, which basically means the opportunity cost of holding money is higher, causes demand for money to decrease which eventually makes the currency to depreciate.

Now, this equity between interest rate in different countries in the real world does not always exist because of failure of the assumptions to hold or otherwise and thus allow traders to avail arbitrage option position. Chinn and Meredith (2004) suggests that interest rate differentials are "biased predictors" of exchange rate movements in the short run, thus, resulting in signs opposite to what theory dictates over a short horizon of 12 months. The authors also note that the theoretical linkage between interest rate differential and exchange rate movement is more apparent over horizons of 5 years or 10 years.

3.2 Purchasing power parity (PPP)

The parity theory of exchange rate, based on law of one price, allows one to estimate what the exchange rate between two currencies would have to be in order for the exchange to be on par with the purchasing power of the two countries' currencies over the same basket of good. There are two different versions to the theory-popularized by Gustav Cassel-real exchange rate is considered to be 1 in the absolute version and in the relative version there is no expected movement in real exchange rate and changes in the exchange rate is equal to changes in relative national price levels. Let p* be logarithm of price level in India; p be logarithm of price level in Bangladesh; ${e'}_t^{Tk/Rupee}$ denote real exchange rate[10];

[10] Real exchange rate is nominal rate adjusted for price level

Absolute version of PPP $e{\prime}_t^{Tk/Rupee} = e_t^{Tk/Rupee}(p - p*)$

Therefore, $e_t^{Tk/Rupee} = p - p*$

Relative version of PPP $\Delta e_t^{Tk/Rupee} = \Delta(p - p*)$

Or, $\Delta e_t^{Tk/Rupee} = (\pi - \pi*)$

To elaborate further, Cassel (1918) points out in simple terms that whatever a basket of goods and service cost in one country, once converted to another currency, one should be able to purchase same level of goods and services given a floating exchange rate system is prevalent in both countries. For example, suppose there is increase in inflation in the domestic country due to monetary disturbance and the domestic basket of good rise in price. If the nominal exchange rate remains the same then this means that foreign residents can no longer buy the same level of goods and services as their currency is undervalued with comparison to domestic currency and the domestic currency on the same count is overvalued. In such a situation, according to PPP theory, the nominal exchange rate must go through an adjustment process. With the domestic currency being overvalued, it makes foreign goods cheaper which induces domestic residents to buy goods from abroad. This increases the supply of domestic currency and at the same time puts an upward pressure on demand for foreign currency which eventually causes foreign currency to *appreciate* and domestic currency to *depreciate* as a mechanism to settle down to PPP exchange rate.

One practical implication of PPP is the Big Mac index (Economist, 1986) which is basically used to see if nominal exchange rate of a country is undervalued or overvalued compared to the price of a Big Mac, a popular hamburger served at fast food chain MacDonald's. Let's say if the price of a Big Mac in US is $4.7 while in China it is about 2.7 yen then we can say that yen is undervalued and there is pressure on yen to appreciate in value to rise up to the PPP exchange rate.

Data, however, rejects this hypothesis which leads us to the PPP puzzle postulated in Rogoff (1996). On theoretical disposition, the fact that PPP deviates in the short run is inevitable given the nature of international transaction that includes trade barriers such as tariffs and also transaction cost. To put it in other words, there is a short-run deviation from PPP owing to these factors. Therefore the PPP debate or the validity of the PPP hypothesis hinges more around its long-run validity in the literature and thus the use of real exchange rate have been more useful to economists. Nonlinear modeling of real exchange rate with smooth version of threshold autoregressive[11] resolves the PPP puzzle as can be seen in the work of Taylor, Peel and Sarno(2001) who find real exchange rate data to be nonlinearly mean reverting and the half-life to be much less, particularly under 3 years, thus, favoring PPP evidence in the long-run.

In the forecasting literature, the general consensus is that PPP forecasts well for long-horizon not short horizon (Ching, Chinn and Pascual, 2005; Engel, Mark and West, 2007). However, one strand of recent forecasting literature also gives positive implication for accounting for smooth nonlinearities in nominal exchange rate data that result in PPP as a better exchange rate predictor. For example, Suarez and Lopez (2010) employs smooth transition error correction model (STEC)[12] on panel data of nominal exchange rate and CPI levels and finds the model to beat random walk specification at both short and long horizon and over different forecast windows, implying robustness of the out-of-sample results. model for nominal exchange rate forecasting. On a slightly different note, Bjornland and Hungnes (2006) stresses that importance of interest rates in forecasting exchange is substantial- a structural model combining both PPP fundamentals and interest rate differential beat random walk rate-out-of-sample.

3.3 Monetary theory of exchange rate determination

The monetary approach to exchange rate determination, as the name suggests, puts emphasis on the money market-through demand for and supply of money-as a way of determining exchange rate. Central to a monetary model is the money demand function and three crucial assumptions following the flexible version of the monetary approach (also known as Frenkel and Bilson (1978)) which are as follows:

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[11] According to Taylor and Taylor(2004), STAR models address the goods aggregation problem-as transaction costs are different for different goods, more and more thresholds will be breached and the speed of adjustment, as a result will also vary across goods

[12] On the other hand, STEC model

- Prices are completely flexible, i.e., the aggregate supply curve is vertical
- Demand for money is as follows: $M^D = K\ P\ Y = K\ I$; where I is the nominal level of income
- PPP always holds, i.e., $p = ep *$, where p is domestic price level and p* is the foreign price level

According to Frankel (1983), at money market equilibrium, the money demand function for the domestic country is as follows:

$$m = p + \O y - ƛi$$

And, similarly, the money demand function for the foreign country will be:

$$m^* = p^* + \O y^* - ƛi^*$$

A relative money demand function is derived by taking difference of the two equations:

$$m - m^* = p - p^* + \O(y - y^*) - ƛ(i - i^*)$$

The monetary model mentioned above assumes that PPP holds at all times which means the following condition holds:

$$e = p - p *$$

Solving for p-p* in equation (5) gives us the following equation:

$$p - p^* = (m - m^*) - \O(y - y^*) + ƛ(i - i^*)$$

Substitution equation (6) in equation (5) is what gives us the fundamental equation of the monetary model as presented below:

$$e = (m - m^*) - \O(y - y^*) + ƛ(i - i^*)$$

In the face of exogenous shocks, the predictions of the model are as follows- if money stock increases (all other exogenous variables remaining same), there is an increase in excess supply of money at all price levels. This excess supply of money in the economy implies an excess demand for goods and services. Since output is fixed in this model, the excess demand for goods drive price upwards in the domestic prices level. This in turn, results in the domestic economy to

become under competitive. So for PPP to hold, domestic currency must *depreciate* in terms of the foreign currency.
Let us now consider the exogenous shock of an increase in the level of income. At $p_{0,}$ there is an excess demand for money which implies an excess supply of goods and services. This leads to a fall in the domestic price level and therefore the domestic economy becomes more competitive and hence an appreciation of the currency is required so that PPP holds. From this we can deduce that, in a monetary model, an increase in real income leads to an *appreciation* of the domestic currency.

Lastly, an increase in price level will see an increase in the slope of the PPP curve. At the original nominal exchange rate, the domestic economy becomes over competitive; hence the domestic currency must appreciate. To summarize, according to monetary model prediction, a rise in the foreign price level leads to an *appreciation* of the domestic currency and vice versa.

Simultaneously, if we relax the assumption of PPP but let UIP hold, which seemed to be the scenario post floating regime leading to volatile nature in the exchange rate, we have the *sticky price version* of the monetary model(aggregate supply curve is vertical) developed by Dornbusch. This version assumes instead that prices are sticky in the short run and hence an initial increase in interest rate (let's say due to reduction of money supply in the economy) would cause capital inflow that would result in *appreciation*[13] of the nominal exchange rate. In other words, the model predicts a negative coefficient for interest rate differential in the short-run and a positive coefficient in the long-run where PPP hold.

Based on the monetary theory of exchange rate determination, the study applies the following sticky-price variant of the monetary exchange rate model specification with a trade balance variable augmented in it:

$$e_t = (m - m *) + (i - i *) + (\pi - \pi *) + (TB)$$[14]

Where, $\boldsymbol{e_t}$ is the log of Taka/Rupee exchange rate; asterisks denote variables for India; $(\boldsymbol{m} - \boldsymbol{m} *)$is the money supply differential; $(\boldsymbol{i} - \boldsymbol{i} *)$ is the interest rate differential and$(\boldsymbol{\pi} - \boldsymbol{\pi} *)$ is the inflation differential between the two countries. A trade balance variable, $\boldsymbol{TB}$ is augmented

[13] This appreciation of the exchange rate is termed as "overshooting" in the literature , an appreciation that is beyond the PPP level
[14] The study does not consider any specific variant of the monetary models of exchange rate determination because of lack of monthly data on income for Bangladesh. However, the model can be considered close to the sticky price version of the monetary model as it uses inflation differential as one of the regressors

in the standard monetary exchange rate model to see if there's any impact of the existing and much debated trade imbalance between India and Bangladesh on the bilateral exchange rate of Taka/Rupee. It should be noted that Hooper and Morton (1982) was the first to incorporate a trade balance variable in the monetary exchange rate model but the authors did so with an intention to use it as a broad substitute for stock of balances, what came to be known as the portfolio balance determination of exchange rate in the literature. The study here uses trade balance only as the difference between import and export between the two countries, not as a relative change in trade balance, i.e., TB-TB* as used in Hooper and Morton (1982).

4. Data

In order to conduct the estimation, the study uses monthly data from May 2003 to March 2016 for India and Bangladesh International Financial Statistics (IFS) databsase. The time period is chosen from May 2003 onwards to reflect the shift to floating exchange rate regime by Bangladesh Bank on the same year. All the estimation is performed in statistical software STATA.

The dependent variable, nominal exchange rate is quoted as units of domestic currency per foreign currency, i.e., Bangladesh taka per Indian Rupee, and is made to undergo log transformation and labeled as "ltk_rup". Relative changes in monetary fundamentals such as Indian money supply and Bangladeshi money supply, m1 and m2 (in million USD), are labeled as "m1g_gap_yoy" and "m2g_gap_yoy" respectively and calculated as the percentage change during the last twelve months in the domestic country against the percentage change in the last twelve months in the foreign country and can be represented as follows:

$$\Delta m_t = \frac{m_t^{domestic} - m_{t-12}^{domestic}}{m_{t-12}^{domestic}} - \frac{m_t^{foreign} - m_{t-12}^{foreign}}{m_{t-12}^{foreign}}$$ [15]

The relative change in CPI inflation, labeled as "inf_gap_yoy" (in %) is also calculated in the similar way as shown in the equation above. The variables are calculated this way to avoid any seasonal effects at an annual lag in the data and thus ensure variance stability necessary to conduct the analysis. The interest rate gap, labeled as "int_gap", is a growth variable in itself and is simply calculated by taking the difference between Bangladesh Bank's deposit rate and India's 10-year government securities rate. An alternative to this variable is considered by taking the difference between Bangladesh stock exchange (BSE) equity return and Indian Stock Exchange(BSE) equity return, labeled as "eqret_gap"(in %).Finally, the trade balance variable augmented in the framework is calculated as the difference between Bangladesh import to India (in million USD) and Indian export to Bangladesh (in million USD) as recorded in Bangladesh's current account.

[15] The month-on-month change of the same variables are calculated using a lag of one month: $\Delta m_t = \frac{m_t^{domestic} - m_{t-1}^{domestic}}{m_{t-1}^{domestic}} - \frac{m_t^{foreign} - m_{t-1}^{foreign}}{m_{t-1}^{foreign}}$

5. Methodology

5.1 Basic statistical issues in nonlinear time series analysis

We begin this section by establishing some of the salient features of non-linear time series and into matters pertaining to detection of it and selection of appropriate test statistics. Next, we move onto the second part of our methodology which entails a discussion of the Markov switch model employed for our parameters. The third sub-section carries out a discussion of various forecasting schemes, test procedures employed in this study to decide the better competing model.

5.1.1 The notion of unit root

The term unit root is synonymous to non-stationary, a property that embodies a general tendency of variables to increase over time. Statistically speaking, this would mean that the mean, variance and covariance of the series are all time-dependent. In presence of unit root, spurious regression, a term coined by Granger and Newbold (1974) becomes prevalent, i.e., there might not be any meaningful relationship between the regressor and the regressant but estimates might still be highly statistically significant or vice versa. A simple example of non-stationary time series is a random walk as in the following equation:

$$y_t = \rho y_{t-1} + \varepsilon_t \text{[16]}$$

Unit root tests, therefore, test the following null hypothesis of ρ equal to 1 to alternative hypothesis of $\rho<1$.Unit root in data is cured by taking first differences[17] of the time series in case of random walk without drift or pure random walk. The following equation for series y_t illustrates difference stationary process:

$$\Delta y_t = y_t - y_{t-1} = \varepsilon_t$$

[16] Here e_t is a white noise error term with zero mean and constant variance, I.e. stationary

[17] There can be instances where time series data requires second differencing or third differencing depending on the number of unit root present in the data, hence, a series is said to integrated to the order of 1,i.e., I(1) if it has been differenced once or integrated to the order of n, I(n), if differenced n times.

Certain types of time-series data might also be a trend stationary[18] process. This is true for data in which the mean is not constant but variance is and hence requires the deterministic trend to be removed by the process of de-trending, i.e., subtraction of the mean of y_t from y_t . The following equation represents a trend stationary process:

$$\Delta y_t = \beta_1 + \beta_2 t = \varepsilon_t$$

5.1.2 The notion of nonlinearity

To understand the idea of nonlinear process in time series, let us first consider what linear adjustment could mean. Simply put a linear equation has variables raised to the power of one and demonstrates constant coefficients. For example, consider a situation where investment is a constant proportion of investment as in the following equation:

$$i_t = \beta(c_t - c_{t-1}) + e_t$$

On other hand, non linear variables are variables that grow exponentially. Most time series data exhibit nonlinear dynamics in its data generating process and perhaps they do so in a more complex manner than the aforementioned examples of nonlinearity. Variables such as exchange rate, stock prices have undergone structural changes and as a result have prompted researchers to abandon typical linear difference equation models and drift towards the use of nonlinear models to explain nonlinear behavior in time series. For example, Hamilton (1989) first used nonlinear model to demonstrate the cyclical behavior of booms and bust in U.S. output growth.

Regime switching model is one class of nonlinear models where the particular value of the dependent variable depends on the state of the system which is a state of dynamic equilibrium[19]. To illustrate better, consider the role of certain economic variables such as unemployment rate-during recession, unemployment rate is more likely to show upward adjustment than downward. The simplest example of a regime switching model is a threshold autoregressive

[18] The distinction between a unit root process and a trend stationary process is that the former has non-mean reverting properties while the latter possess mean-reverting properties. Both are non-stationary processes.

[19]

(TAR) that has the capacity to capture jump and asymmetric characteristics pioneered by Tong (1983) and represented by the following equation:

$$y_t = \begin{cases} \rho_1 y_{t-1} + \varepsilon_{1t} \ if \ y_{t-1} > 0 \\ \rho_2 y_{t-1} + \ \varepsilon_{2t} \ if \ y_{t-1} \leq 0 \end{cases}$$

where , yt-1=0 is a threshold and two different autoregressive processes run around this threshold at two different speed of adjustment, rho1 and rho2. The above equation manifests piecewise differential equation- each autoregressive process is linear on its own but together, the system of equation is nonlinear-the cornerstone of TAR type models. As noted by Tong (2010), "the steady state for the dynamics inside each regime (i.e. above or below zero in this case) is a limit point; yet by dividing the state space into two regimes, each regime being governed by different simple linear dynamics, a new steady state of a fundamentally different character can be created. That is the magic of nonlinearity!"

5.1.3 Tests/detection of unit root

Standard linear unit root tests such as Augmented Dickey-Fuller perform poorly in presence of nonlinearities and structural break in time series data and hence such data demands a special need for non linear unit root test. To address this issue, we employ two types of unit root tests, namely Dickey Fuller Generalized Least Squares (DF-GLS) and Kwiatowaski-Phillips-Schmidt-Shin (KPSS). These tests complement one another as such that if one test accepts unit root hypothesis and the other test rejects it (due to different type of argument in the null hypothesis), one can obtain a fairer verdict on the actual presence of unit root in each of the series of data used in the study.

DF-GLS is the modified version of Augmented Dickey Fuller(ADF) test which removes the deterministic trend from the data by generalized least square method(hence the name) before applying the Dickey-Fuller test on the residuals of the regression. As a result, the test is known to be more efficient in terms of power compared to the ADF test when trend is present in the data and delivers same result when no such unknown trend or mean is present. DFGLS test statistics has the similar asymptotic null distribution like the ADF test and hence the same critical values can be used for both the test. The main improvement of DFGLS over ADF is that the former allows for higher autoregressive process in its model specification to resolve the issue of serial correlation in residuals. Let us consider the following model for series y_t :

$$\Delta y_t = \alpha + \omega y_{t-1} + \delta \Delta y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} \epsilon_t$$

Therefore, the null and alternative hypotheses of the test are as follows: $H_0: \omega = 0$; $H_{A:}\ \omega < 0$. The test statistics follow a tau (τ) distribution and the values from the test statistic are calculated as follows:

$$DF(\tau) = \omega^{\wedge}/SE(\omega^{\wedge})$$

Where, $\omega^{\wedge}$ is the coefficient estimate of ω and $SE(\omega^{\wedge})$ is the standard error of $\omega^{\wedge}$. The inferences of the test of the hypotheses are drawn by comparing the values of the test statistic with the critical values- if the test statistic value is smaller than the critical value, then the null hypothesis of presence of unit root is rejected and vice versa.

KPSS test, on the other hand, tests different null and alternative hypothesis compared to most other unit root test. The null hypothesis in this test, instead, tests if the test statistic rejects trend stationarity and the alternative hypothesis supports presence of unit root. The test statistic can be represented as follows:

$$KPSS = \sum_{t=1}^{T} S^2 / s^2 T^2$$

Where, T is the sample size; $\boldsymbol{s^2}$ is the Newey-West estimate of the long-run variance; $\boldsymbol{S^2}$ is the partial sum of residuals.

5.2 Markov switching-The model

Markov switching model is one type of regime switching model that is typically applied on data sets to capture nonlinearity in the data generating process as such that the model can capture asymmetric behavior of the data across different regime or subsamples. For example, variables like GDP go through phases of booms and recession, exhibiting varying characteristic across each state. The specification makes regime switching occur by allowing the mean and the variance of the data to switch across multiple states being dependent upon a non-observable state variable, s_t .In our case, exchange rate dynamics is analyzed by allowing the level of exchange rate to alternate between prolonged periods of appreciation and depreciation. The simple representation of the standard two-state Markov switching model is as follows:

$$e_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t \quad \varepsilon_t \sim N(0,1)$$

Alternatively, the 2-state Markov switch model can also be represented as the following system of equation:

$$e_t = \begin{cases} \mu_1 + \sigma_1\,\varepsilon_t \; if \; s_t = 1 \\ \mu_2 + \sigma_2\varepsilon_t \; if \; s_t = 2 \end{cases}$$

where, the unobserved state variable s_t takes on values in the set {1, 2} and ε_t is the white noise error term; μ is the sample mean and σ is the variance of the error term . The evolution of the state variable from one regime to another, in turn, depends upon what is known as the transitional probabilities, i.e., the probability of being in each state, which follows a first-order Markov process. The process is known as Markov process as the timing of the switching is unknown and hence the occurrence of each state is random and solely depends on the recently prevailed state, i.e. , $P(s_t = 2|s_{t-1} = 1) = p_{12}$, where, $p_{11+}p_{22} = 1$.

The model draws interesting and useful inferences about the transition probabilities and other model parameters. For example, if probability of being a certain state is high or close to 1 then that shows persistence on behalf of the data. The coefficient of mean, on the other hand, gives an estimate of the trend of the data- a positive and a statistically significant mean represents uptrend in the data or depreciation of exchange rate and a negative and a significant means a downtrend in the data or appreciation of the exchange rate. The sigma gives inference about the volatility shifts in the data.

To analyze the role of monetary fundamentals on exchange rate dynamics, we also include fundamentals in our Markov switching model in mean and variance which can be represented as follows:

$$e_t = \mu_{s_t} + \beta X_t + \sigma_{s_t}\varepsilon_t \quad \varepsilon_t \sim N(o,1)$$

Where, β is kept constant across the two states and represents a vector of exogenous variables such as money differential, interest and inflation differential and trade balance.

5.3 Forecasting

5.3.1 Key methodological factors in forecasting exchange rates

Other than selection of *model* and *macroeconomic predictors*, what's equally important for a successful exchange rate forecast is the choice of *forecasting schemes*, *forecast horizon* and the *evaluation methods* to judge the forecast results. It's this lack of robustness to aforementioned criterion that makes exchange rate forecasting a daunting task. The literature on this end is abundant with contradicting results and exchange rate forecasting is seen to vary over different time periods and also vary depending on whether they are in-sample or out-of-sample. Rossi (2013) notes that certain predictors forecast well in-sample while others forecast better out-of-sample. The author further stresses the importance of forecast horizon, h, and that a predictor's predictive ability of exchange rate also depends on it. In Chin, Cheung and Pascual's 1995 paper we see that monetary fundamentals fail at making prediction at short-horizon, i.e., one-month-ahead prediction, while Mark (1995) at the same time show evidence for monetary fundamentals' long-horizon predictability at 3 to 4 year horizon. Rossi (2013) also adds that exchange rate forecasting is neither robust to the forecast sample or the out-of-sample forecast period that is used for forecast evaluation. In this regard, the author cites the work of Giacommini and Rossi(2010) who find that the forecastability of UIP and Taylor rule fundamentals change over different out-of-sample periods with respect to a random walk benchmark in each case.

The importance of the choice of a benchmark model was also emphasized by the author who notes that the random walk without drift is the consistent benchmark model for exchange rate forecasting all throughout the literature. The most common forecast methodologies used in the literature are rolling and recursive forecast schemes. The rolling window size varies across papers but typically ranges from 50 to 120 for monetary fundamentals. For evaluating forecast, the three typical loss functions that are used are mean square error (MSE), root mean square error (RMSE) and also mean average error (MAE). Alternatively, another forecast evaluation tool is the direction of change statistic, a statistic that calculates the sign of change in forecast of exchange rate (e.g. Engel (1994)). To assess the significance of forecast performance, the author makes distinction between out-of-sample" absolute" and "relative"[20] tests of forecast accuracy,

[20] The relative tests of forecast accuracy in exchange rate literature refer to Diebold and Mariano, Clark and West and ENC-NEW test procedures etc.

noting that the former is useful for measurement of optimal forecast and the latter for the purpose of evaluating which forecast is best among the competing models with RW as the benchmark model.

5.3.2 Forecasting schemes

Static vs. dynamic window

Static forecasting as the name suggests assumes that the world remains the same and makes use of actual data to forecast out-of-estimation, one-step-ahead errors. This basically means that once forecast errors are generated it can be compared with real world data , which is available to the user, to judge the performance of the model. This type of forecasting is known as the ex-post forecast. Assuming the world is static, the forecasting scheme does not include any lagged dependent variable. Let us consider the concept in a linear regression framework:

$$y_t = X_t \beta_t + \varepsilon_t$$

Dynamic forecasting, on the other hand, is quite opposite to static forecasting in the sense that it produces ex-ante forecast where actual values outside the estimation period are not available to the modeler. Another way to distinguish it from static forecasting is the fact that it incorporates a lagged dependent variable in its specification as follows:

$$y_t = X_t \beta_t + y_{t-1} + \varepsilon_t$$

Rolling vs. recursive window

Rolling forecasting is conducted by using the most recent observation, say n, and thus progressively moves forward over time, t. A simple representation of a rolling regression would be as follows:

$$y_t(n) = X_t(n) \beta_t(n) + \varepsilon_t(n)$$

In a rolling window regression the most recent observations are used in a add-and- drop manner. For example, if the in-sample portion[21] is from 1998:01 to 2008:01 and the out-of-sample portion is 2008:02 to 2016:01, then as a first step the model will estimate in-sample using data from 1998:01 to 2008:01 to forecast 2008:02. Next, the forecasting model will drop the first observation and add the recently projected value to re-estimate the model by using observation from 1998:02 to 2008:02 to forecast 2008:03 and so on. It is because of this add and drop process, rolling regression retains a fixed window of data each time to re-estimate the model and predict future values.

Recursive approach instead uses an increasing window of data to predict future values and as such makes use of all past observations. For example, given the same aforementioned in-sample and out-of-sample periods, this approach will as a first step estimate the model in-sample by using data from 1998:01 to 2008:01 to forecast 2008:02 and in the next step, it will re-estimate model parameters from 1998:01 to 2008:02 to forecast 2008:03 and etc.

5.3.3 Forecast evaluation method

Through our comparison of forecast errors between the two competing models in, we address one of our key research questions- do macroeconomic fundamentals, that are of great theoretical interest to researchers, determine the future path of exchange rate or is the alternative of no exchange rate predictability or a random walk model forecast is just as good or better. To this end, we employ two loss functions and Diebold Mariano test statistics to derive the forecast accuracy results.

Comparing statistical measure of forecast accuracy

Central to the idea of forecast accuracy measurement is a loss function, L (e). So basically, when any forecast is produced one wishes to assess the expected loss associated with each of the forecast- higher the loss, lower the accuracy and vice versa. Typically, forecast accuracy is evaluated by using the one-step-ahead, out-of-sample forecast error[22] and the sum of the

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[21] The first step to a successful forecasting is to divide the data into two segments- the "fitting segment" or the in-sample portion and the "forecasting segment" or the out-of-sample portion (Montgomery, Kulhaci and Jennings, 2008).

[22] The distinction between residual and the forecast error is that the former is the difference between the observed and the fitted value ,i.e., it arises from the model-fitting process while the latter is the error made while forecasting the variable/variables of interest

squared forecast error[23] is one such loss function that is widely used in the literature to gauge statistical superiority of competitive models. It is represented as follows:

$$MSE = \sigma_{e(1)}^{2} = \frac{1}{n}\sum_{t=1}^{n}[e_t\,(1)]^2$$

We also apply another loss function, namely MAE, to check robustness of our results across different loss criteria. It can be represented as follows:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}[e_t\,(1)]$$

Testing for equal predictive ability

To compare MSE's across the two models, we simultaneously employ forecast evaluation technique of testing for equal predictive ability between two forecasts by carrying out minimum mean square forecast error test- also termed as the "MSE dominance" approach in the literature- as a way of determining the predictive content of monetary fundamentals to exchange rate. The head-to-head test of forecast accuracy measures of each of the model, based on a loss differential function, is considered a better approach to forecast evaluation that uses MSE on a stand-alone basis, according to empirical evidence.

In hypothesis testing terms, we are basically testing the following accuracy hypothesis (Diebold, 2007):

$$E\left[L\left(e_{t+h,t}^{MS}\right)\right] = E\left[L\left(e_{t+h,t}^{RW}\right)\right];$$

Against the alternative that the one or the other is better, i.e.,

$$E\left[L\left(e_{t+h,t}^{MS}\right)\right] > E\left[L\left(e_{t+h,t}^{RW}\right)\right] \text{ or, vice versa}$$ [24]

Equivalently, what the equal predictive ability hypotheses above tell us is that the expected loss differential from the models is zero:

$$E(d_t) = E[L\left(e_{t+h,t}^{MS}\right) - E\left[l\left(e_{t+h,t}^{RW}\right)\right] = 0$$

[23] By applying sum of the squared forecast error or MSE, one assumes a quadratic loss function of the form, $L = e^2$

[24]

The study uses Diebold and Mariano (1995) test statistic to compare predictive accuracy across models. The DM test statistic is asymptotically normally distributed under the null hypothesis of no difference in MSE or MAE. The inference from the test statistic is interpreted as follows: if the DM test statistic falls outside the range of the critical values or are too extreme, the null hypothesis of no difference will be rejected. That is, $|\mathrm{DM}| > Z_{\alpha/2}$, where $Z_{\alpha/2}$ is standard z-value from standard normal table and α refers to desired level of significance.

6. Results

6.1 Descriptive statistics

TABLE 1. Descriptive statistics

	Mean	Std. Dev	CV	Min	Max
$\boldsymbol{e_t}$	1.44	0.15	0.11	1.15	1.74
$\boldsymbol{\pi_t - \pi_t^*}$	0.02	3.39	144.55	-8.43	6.68
$\boldsymbol{i_t - i_t^*}$	0.31	1.63	5.33	-2.51	4.39
$\boldsymbol{m1_{gt} - m1_{gt}^*}$	1.73	11.27	6.51	-23.83	28.39
$\boldsymbol{m2_{gt} - m2_{gt}^*}$	2.82	12.83	4.55	-21.87	34.13
$\boldsymbol{r_t - r_t^*}$	-0.20	10.17	-51.35	-36.84	25.37
$\boldsymbol{TB_t}$ $(\boldsymbol{US}\$)$	263.12	141.64	0.54	103.17	657.92

Table 1 reports descriptive statistics for the following variables- $e_t$the monthly levels of taka/rupee exchange rate and a set of fundamentals such as $\pi_t - \pi_t^*$, the percentage difference between foreign and domestic CPI inflation; $i_t - i_t^*$, the difference in domestic and foreign interest rates; $m1_{gt} - m1_{gt}^*$, the percentage difference between domestic m1 money supply growth and foreign m1 money growth; similarly, $m2_{gt} - m2_{gt}^*$, the difference between domestic m2 money growth and foreign m2 money growth; $r_t - r_t^*$, the difference between domestic stock exchange equity return and foreign stock exchange equity return; and the difference between import and export trade figure with foreign country, TB_t. The mean values for each of the series are moderate but the mean value for TB_t is quite high. The standard deviation for e_t, $\pi_t - \pi_t^*$, $i_t - i_t^*$ are somewhat close to unity and on the other hand, $m1_{gt} - m1_{gt}^*$, $m2_{gt} - m2_{gt}^*$, $r_t - r_t^*$, TB_t demonstrate very high standard deviation. The coefficient of variation **(**CV**),** which is the ratio between standard deviation and the mean, indicate $\pi_t - \pi_t^*$ and $r_t - r_t^*$ series as highly volatile in terms of magnitude. The difference between min and max values, which is a measure of variability, are different for each fundamental.

6.2 Stationarity test

TABLE 2. Unit root test

	DF-GLS	KPSS
$\boldsymbol{e_t}$	-1.129	0.426**
$\boldsymbol{\pi_t - \pi_t^*}$	-2.482**	0.460**
$\boldsymbol{i_t - i_t^*}$	-1.771	0.127**
$\boldsymbol{m1_{gt} - m1_{gt}^*}$	-0.960	0.084**
$\boldsymbol{m2_{gt} - m2_{gt}^*}$	-1.343	0.072**
$\boldsymbol{r_t - r_t^*}$	-1.950	0.074**
$\boldsymbol{TB_t\ (US\$)}$	0.082	0.196

After sufficient adjustment and transformation of our raw data, we conduct unit root test on each of our data series as stated in the Table 2. At first, we conduct Dickey-Fuller Generalised Least Square (DF-GLS) test (without trend) by Elliot, Rothenburg and Stock(1992) up to 13 lags and find that most of our variables are non-stationary at 5% significance level except for inflation rate differential($\boldsymbol{\pi_t - \pi_t^*}$). Hence, we also carry out Kwiatowski et. al (1992) KPSS test (with constant only) and find out that almost all variables are statistically significant at conventional levels of significance. However, trade balance ($\boldsymbol{TB_t\ (US\$)}$) still remains a non-stationary process which is why we use a logarithmic transformation of the series in our analysis. In conclusion, by using inference from both of these tests, it can be deduced that all our variables are stationary. For more evidence on variance stability see Appendix A.

6.3 Parameter estimates: Regime switching in variance only

TABLE 3. Parameter estimates for Markov switch model with variance switch

	TK/RUP					
	E&H	PPP	IRP	PPP+IRP	Monetary model	Monetary+TB
State1						
μ	1.259***	1.246***	1.263***	1.246***	1.284***	1.246***
σ	0.046	0.046	0.047	0.046	0.080	0.079
State 2						
μ	1.533***	1.539***	1.533***	1.539***	1.573***	1.542***
σ	0.097	0.082	0.095	0.081	0.048	0.046
$\text{Inf} - Inf^*$	–	0.011***	–	0.011***	0.012***	0.012***
$\text{Int} - Int^*$	–	–	- 0.004	0.0004	-0.006	-0.0097***
$\text{m} - m^*$	–	–	–	–	-0.004***	-0.004***
Log(TB)	–	–	–	–	–	0.044***
P_{11}	0.9890	0.9890	0.9891	0.9891	0.9905	0.9905
P_{22}	0.9859	0.9859	0.9859	0.9859	0.9843	0.9844
P_{12}						
P_{21}						
Log-likelihood	171.523	189.155	171.9	189.41	205.943	210.691
RCM	7.179	6.785	7.105	6.820	7.554	7.273
SBIC	-2.108	-2.213	-1.99	-2.184	-2.364	-2.393
Expected duration of the regimes(in months)						
Regime 1	91.6	91.7	91.7	91.7	105.8	105.7
Regime 2	71.2	71.2	71.1	71.1	63.9	63.9

Note: Asterisks refer to the level of significance-*:10%, **:5%, ***:1%

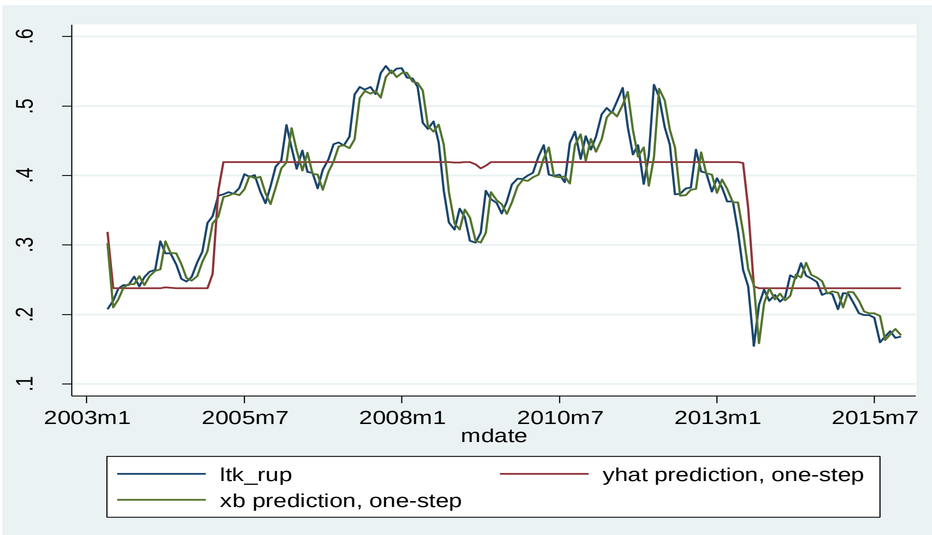
For parameter estimation, we fit our exchange rate data to nonlinear Markov switching dynamic regression (MS-DR) which switches the drift parameter or the intercept, μ, by default. Through this methodology we consider a couple of structural models of exchange rate determination and test (a) whether these models provide useful explanation to exchange rate movement of Taka/Rupee with nonlinear structure in the data being taken into account; (b) if the MS process evolves differently in each state and if macroeconomic fundamentals play a role in driving this switching process. At first, we consider the Markov switch model with state-dependent or switching mean and variance. Table 4 above report the results. State 1 with the lower level of positive and statistically significant μ across all models in the study is the "appreciation" regime and state 2 with the higher level of the mean is consequently the "depreciation" regime. Both state 1 and state 2 is characterized by high estimated probabilities in MS-DR model ranging from about 96% to as high as 99% which indicates that the states are highly persistent. Putting it another way, the probabilities from switching from one regime to another is extremely low-as low as 0.01. This also means that the expected durations of state 1 and state 2 are relatively high-as high as 5 to 8 years.

Turning to the estimated coefficients of each of the models, we begin our analysis with PPP and IRP model-two of the assumptions or applications of monetary exchange rate model. In case of PPP, while testing whether inflation differential moves in tandem with exchange rate movement, we find positive and statistically significant coefficient, i.e., higher the inflation rate differential, the exchange rate increases which corresponds to a depreciation of the domestic currency relative to the Indian Rupee. The result is important as we can conclude that PPP holds for the particular data set and at the same time, highlights the importance of nonlinear relationship between inflation differential and exchange rate (See Grauwe and Vaersteekiste (2001) for reference). In case of IRP model, we find the interest rate differential has a negative, much less than unity in magnitude and statistically insignificant impact on exchange rate of Taka/Rupee. This particular result demonstrates a sign that is aligned with the standard monetary theory and at the same time deviates from the theory. If we consider short-run effects of interest rate differential on exchange rate then we have theoretically coherent sign, i.e., in the short run prices are sticky and hence interest rate differential has a negative coefficient by virtue of exchange rate overshooting. If we, however, consider the long-run effects, then our result deviates from theory as the sign should instead be positive as in the long-run PPP effects hold. The combined effect of interest gap and inflation gap on exchange rate process is what stands for in the" PPP+IRP" model in the Table 4. The result in this is the same as it is for the individual cases of the exchange rate identities-inflation gap has positive and statistically significant coefficient while interest gap has the correct signs according to theory but regardless, it is statistically insignificant. According to the RCM values of the these three models just discussed, the Markov process fits the PPP model the best with the lowest RCM value and the IRP model is rendered with the poorest fit as it generates the largest RCM. It must also be noted that for these three models, there is a distinction for variance or sigma (standard deviation) as high volatility and low volatility-state 2 or the depreciating regime has the magnitude as twice as the appreciating regime. However, this conclusion can be drawn only based on the relative strength of magnitude and nothing can be said about the statistical significance of these reported sigma values.

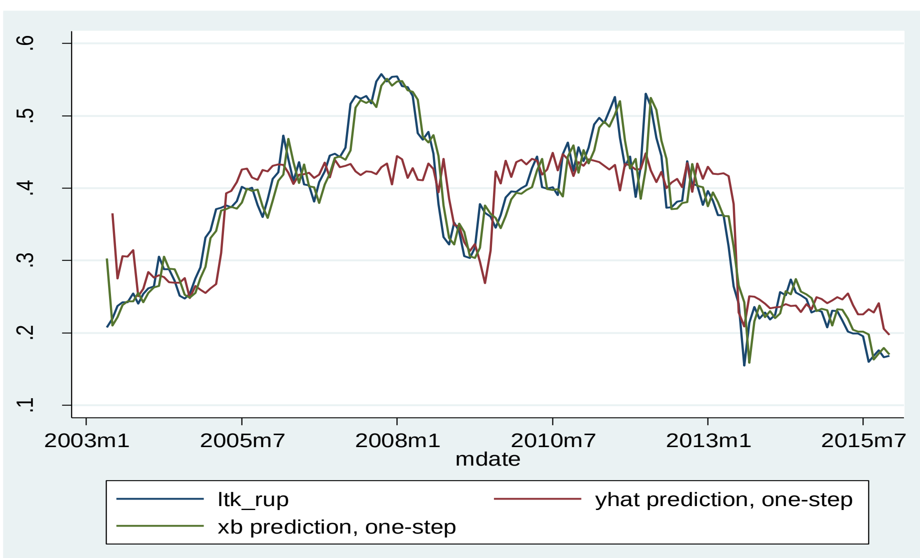
Moving on, we turn our attention to the monetary class of models named as ‘monetary model' (growth gap of m1 money is taken) and ‘monetary+tb' model, where a trade balance, or rather a trade imbalance variable between Bangladesh and India is augmented. Here we see that across all three models, the coefficient of money gap has statistically significant but negative impact on exchange rate. In other words, exchange rate falls, hence, domestic currency appreciates in value. This result does not have correct signs predicted by theory as one would expect domestic currency to depreciate relative to foreign currency to positive changes in money differential to restore money market equilibrium. The negative coefficients on money supply are in line with findings of Beckmann and Czudaj (2015) and Chen (2006). Both of these authors apply a RID variant of the monetary exchange rate model with similar methodology and find signs that are different from theory and therefore conclude that monetary variables are important for exchange rate path even if there is no “theoretically conform impact”. Nevertheless, we cannot ignore the impact of monetary variable on exchange rate and should consider the role of monetary policy on exchange rate determination to be substantial as the variable is highly significant at 1% level. Next, we look at our last model where we find negative but statistically significant variables for interest rate and money gap and a positive and a statistically significant variable for inflation gap. Trade balance also seems to have a statistically significant and a positive impact on exchange rate which is in accordance with theory. According to Investopedia (2015), trade balance affects interest rate by affecting supply and demand for foreign exchange. If a country exports more than its imports then demand for goods in that country generally increases, as a result, demand for that particular currency also increase which in turn causes the price of that currency to rise and the currency to eventually appreciate. And the antithesis occurs when a country gets more imports then export (which is the case for bilateral India-Bangladesh trade): there is depreciating pressure on exchange rate. Hooper and Morton(1989) was the first to incorporate a trade balance variable in the monetary framework but the authors used the variable as a broad substitute for stock of balances between two nations whereas, we simply use the difference between export and import or a trade imbalance variable to see its impact on exchange rate. This model has the lowest RCM among other monetary class of models considered in the study.

6.4 Forecasting

FIGURE 2.



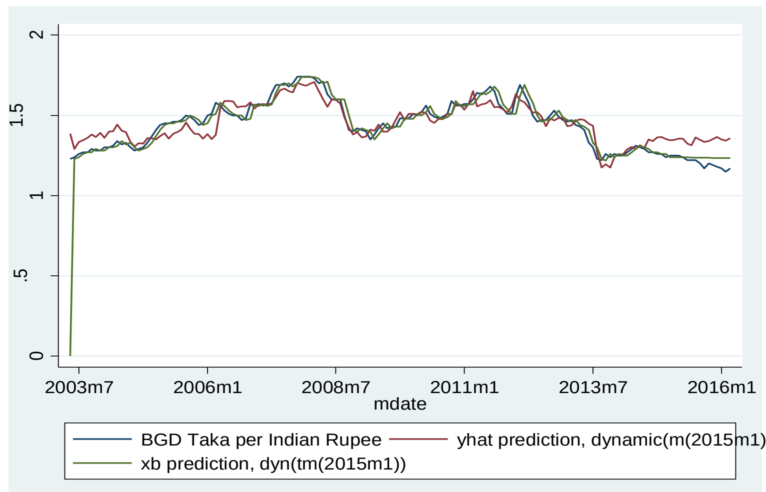
(a)



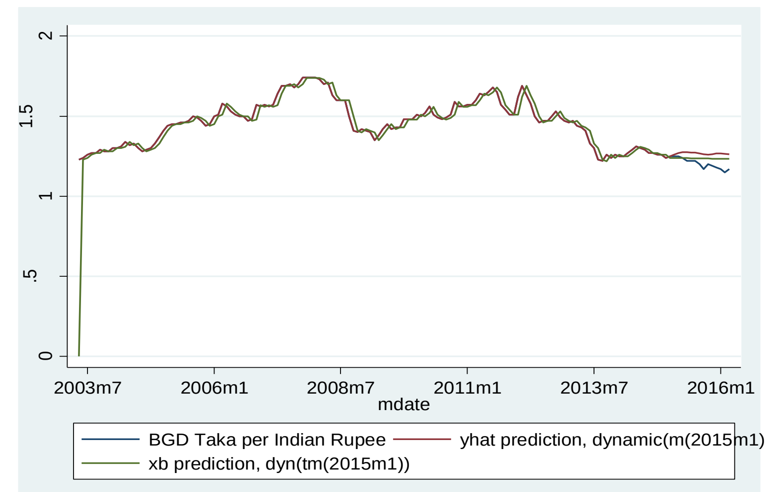
(b)

6.4.1 Static, dynamic, rolling and recursive forecasting

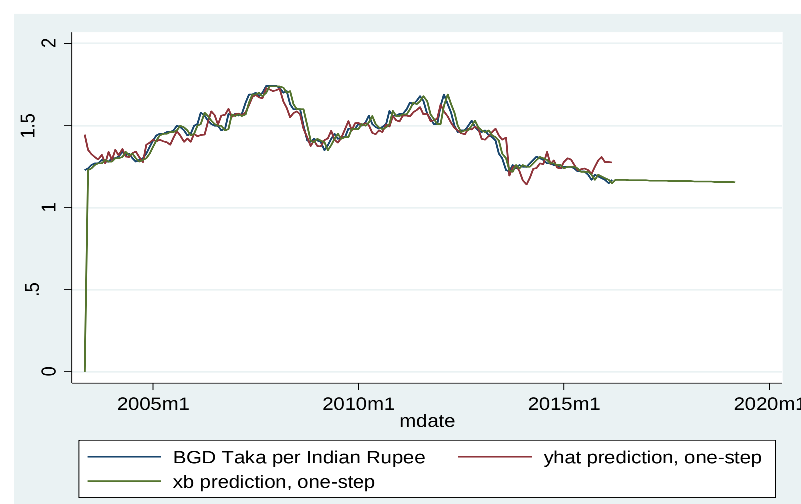
FIGURE 2. Actual and fitted (by RW and MS monetary) bilateral nominal exchange rates (Bangladesh-India)



(a) Static window



(b) Dynamic window



(c) Rolling window

TABLE 4. Static, dynamic, rolling, recursive forecasting

	MSE	**MAE**
	STATIC	
Markov-switch Monetary model	**0.005**	0.058
Random walk model	0.010	**0.033**
Diebold-Mariano test	-0.571	2.38**
	DYNAMIC	
Markov-switch Monetary model	1.183	1.087
Random walk model	**1.170**	**1.078**
Diebold-Mariano test	1.205	1.300
	ROLLING	
Markov-switch Monetary model	**0.084**	**0.258**
Random walk model	0.111	0.301
Diebold-Mariano test	-4.275**	-5.066**
	RECURSIVE	
Markov-switch Monetary model	0.007	0.077
Random walk model	**0.0001**	**0.0004**
Diebold-Mariano test	4.908**	7.6999***

Note: MAE: Mean Average Error; MSE: Mean Squared Error; DM: Diebold Mariano test statistic

Table 4 above reports the forecast errors using two statistical criterion, MSE and MAE, for the two models, MS and RW using four forecast schemes-static, dynamic, rolling and recursive regression. The forecast horizon, h, is one-month-ahead as the study uses monthly data. For static and dynamic forecasting windows, the in-sample or the estimation period is from 2003:05 to 2014:12 and the out-of-estimation or the forecasting period is from 2015:01 to 2016:03. To attain true out-of-sample forecasts, we apply rolling and recursive window to our data. The in-sample period (for the first forecast) is from 2003:05 to 2016:03 and the out-of-sample period is from 2016:04 to 2017:03. The rolling window size of 100 is chosen arbitrarily as recommended in the forecasting literature (see Rossi (2013); Rossi and Innoue (2012)). The significance of the MSE difference between models is assessed by Diebold-Mariano test statistic.

By MSE criteria, MS model with fundamentals has higher forecast accuracy particularly for static regression, in comparison with the benchmark of random walk (RW) model as the former produces lower forecast errors. However, the Diebold Mariano test statistic is not significant in MSE differences. On the other hand, the MAE criterion gives us significant Diebold Mariano test statistic but higher forecast accuracy for RW model. The forecast errors from dynamic regression reject both models in terms of predictive ability due to insignificant DM test statistic. With regard to rolling forecasting window, MS monetary model generates lower forecast error by both MSE and MAE standard. This result indicates significant predictive ability for monetary fundamentals as Diebold Mariano test statistics are significant at 5% level. On the other hand, recursive window produces just the opposite result as to that of rolling- RW model generates lower forecast error and also significant predictive ability in support of the model.

In conclusion, our results from the forecasting exercise seem to be robust across the two statistical criteria but nevertheless, are quite mixed in nature. For out-of-estimation forecast errors using static regression, the results lean towards RW model and thus demonstrate lack of predictive content for monetary fundamentals while the true out-of-sample forecast errors generated from a rolling window are positive in that respect as there is evidence in favor of MS model. This particular result also supports our initial observation of a flat forecast graph in Figure 2. However, this result is not robust to the forecast windows applied as re-estimation of model parameters with a recursive window supports RW model. This piece of empirical evidence makes gauging success of the MS model, at this point, a questionable issue. If we consider the forecasting literature, Rogoff and Stavrakevra (2009) note that predictive ability does considerably depend upon the choice of estimation window in the case of monetary fundamentals.

7. Conclusion

The study applies monetary exchange rate model in a Markov switching framework for the nominal exchange rate of taka/rupee at a monthly frequency from the time period of May 2003-March 2016. We corroborate to the literature by being the first study to test monetary exchange rate model for Bangladesh economy through the nonlinear channel of Markov switching. We also contribute through our forecasting exercise by deriving forecast accuracy results for Markov switch monetary model against the benchmark of random walk, something that has not been considered in the literature before for Bangladesh exchange rate series.

Our study shows strong evidence for nonlinear relationship between monetary fundamentals and exchange rate as we derive highly significant mean values which indicate strong persistence for both appreciation and depreciation regimes. We also find highly significant and theoretically coherent coefficients for interest differential, inflation differential, money growth differential and trade balance. In terms of magnitude of the coefficients, trade balance and inflation differential variable cause the highest amount of impact. For example, a 1% increase in trade balance causes the level of the nominal exchange rate to increase or depreciate by 4%. In our analysis, the money gap variable is the only variable to have signs opposite to that of theory. This anomaly could be due to the fact that m1 money is a scale variable like output gap (omitted due to lack of availability at the desired frequency),hence, causing the former to mimic the latter's appreciating effect on exchange rate. On the whole, these positive results have important implications for the asset markets, to be specific, and the macro economy of Bangladesh in general. While the interpretation of coefficients lends clear cut support for monetary model, our out-of-sample forecasting exercise provides mixed evidence. The results are mostly robust to the statistical criterion of MSE and MAE but not robust to different forecasting windows applied in the study. According to Diebold Mariano test statistic, MSE generated from rolling window supports MS monetary model but, on the other hand, MSE generated from recursive window supports predictive content for random walk. For the parts that our forecasting is bad, one can draw certain implications. As Engel, Mark and West (2007) notes, models like monetary exchange rate are more likely to forecast better with developed country currency than developing country currency as financial markets tend to be more evolved and less regulated in the former than the latter.

Moving forward, there are several scopes to this research that can be exploited. For example, the robustness of our results can be verified across alternative short-run forecast horizons such as 3-month or 6-month horizon. There can also be questions regarding the findings of this research being solely a taka/rupee phenomenon. In other words, would the results vary if we extend the analysis by considering the exchange rate of other trade partners of Bangladesh? Meanwhile, gauging success of Markov switch model is a matter of perspective. At the very least, the study serves as a good riddance from disconnect puzzle of Meese and Rogoff (1989).

8. Bibliography

Basher, S., & Westerlund, J. (2006). Can panel data really improve the predictability of the monetary exchange rate model. *MPRA working paper no.1229* .

Beckmann, J., & Czudaj, R. (2014). Regime shifts and the Canada/Us exchange rate in a multivariate framework. *Economics letters* , 206-211.

Bjornland, H. C., & Hugnes, H. (2006). The importance of exchange rate for forecasting the exchange rate. *Journal of forecasting* , 209-221.

Boughton, J. M. (1988). The monetary approach to exchange rate: What now remains? *Essays in international finance* , no. 171.

Chen, C. (2006). Multiple regime models and exchange rate forecasting. *JCIS* .

Chen, S.-S. (2006). Revisiting the interest rate-exchange rate nexus: a Markov-switching approach. *Journal of development ecoomics* , 208-224.

Cheung, Y.-W. &., Pascual, M. D., & Garcia, A. (2005). Exchange rate model of the nineties: Are any fit to survive? *Journal of international money and finance* , 1150-1175.

Cushman, D. O. (2000). The failure of the monetary exchange rate model for the Canadian-US dollar. *Canadian journal of economics* , 591-603.

Dacco, R., & Satchell, S. (1999). Why do regime switching models forecast so badly. *Journal of forecasting* , 1-16.

Diebold, F. X. (2007). *Elements of forecasting.* Cincinnati: South-Western college publishing.

Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of business and economic statistics* , 253-265.

Dornbusch, R. (1976). Expectations and exchange rate dynamics. *Journal of political economy* , 1161-76.

Enders, W. (2004). *Applied economic time series.* J.Wiley.

Engel, C. (1994). Can the Markov switching model forecast exchange rates. *Journal of international economics* , 151-165.

Engel, C., & Hamilton, J. D. (1989). Long swings in the dollar: Are the in the data and do markets know it. *American economic review* , 689-713.

Engel, Charles; Mark, Nelson C.; West, Kenneth D. (2007). Exchange rate models are not as bad as you think. *NBER working paper series* .

Fisher, I. (1930). *The theory of interest: As determined by impatience to spend income and opportunity to spend it.* New York: the Macmillan co.

Frankel, J. (1983). Monetary and portfolio balance models of exchange rate determination. *Economics working papers, University of Berkeley* , 85-115.

Frankel, J. (1985). The Dazzling Dollar. *Brookings papers on economic activity* , 199-217.

Giacommini, R., & Rossi, B. (2010). Forecast comparisons in unstable environments. *Working paper series no.08-04, Duke University* .

Gournichas, P.-O., & Rey, H. (2007). International financial adjustment. *Journal of political economy* , no.4.

Granger, C., & Newbold, P. (1974). Spurious regressions in econometrics. *Journal of econometrics* , 111-120.

Grauwe, P. D., & Vansteenkiste, I. (2001). Exchange rate and fundamentals: A nonlinear relationship. *CESifo disucssion paper no.577* , Munich.

Hooper, P., & Morton, J. (1982). Fluctuations in the dollar: A model of nominal and real exchange rate determination. *Journal of International Money and Finance* , 39-56.

Hseih, D. A. (1989). Testing for nonlinear dependence in daily foreign exchange rates. *The journal of business* , 339-368.

Investopedia. (2015, April 15). *How does the balance of trade impact currency exchange rates*. Retrieved from Investopedia: http://www.investopedia.com/ask/answers/041515/how-does-balance-trade-impact-currency-exchange-rates.asp

Kaminsky, G. (1993). Is there a Peso problem? Evidence from dollar/pound exchange rate. *American economic review* , 450-72.

Kaufmann, H., Heinen, F., & Sibbertsen, P. (2014). The dynamics of real exchange rates –A reconsideration. *Journal of applied econometrics* , 758-773.

Kilian, L. (1999). Exchange rate and monetary fundamentals: What do we learn from long-horizon regressions? *Journal of applied econometrics* , 491-510.

López-Suárez, C. F., & Rodriguez-Lopez, A. (2011). Nonlinear exchange rate predictability. *Journal of international money and finance* , 877-895.

Mark, N. C. (1995). Exchange rate fundamentals: evidence on long horizon predictability. *American Economic Review* , 201-18.

Mark, N. C., & Sul, D. (2001). Nominal exchange rates and monetary fundamentals: Evidence from a small post-Bretton woods panel. *Journal of international economics* , 29-52.

Mcdonald, R., & Taylor, M. (1994). The monetary model of the exchange rate: long-run relationships, short-run dynamics and how to beat a random walk. *Journal of international money and finance* , 276-290.

Meese, R. A., & Rogoff, K. (1982). Empirical exchange rate model of the seventies: Do they fit out-of-sample. *Journal of international economics* , 3-24.

Meese, R., & Rogoff, K. (1988). Was it real? The exchange rate-interest differential relation over the modern floating-rate period. *Journal of finance* , 933-948.

Michael, F., Mcdonald, R., & Menkhoff, L. (2005). Markov switching regimes in a monetary exchange rate model. *Economic modelling* , 485-502.

Molodtsova, T., & Papell, D. H. (2008). Out-of-sample exchange rate predictability with Taylor Rule fundamentals. *Social science research network* .

Montgomery, D. C., Jennings, C. L., & Kulahci, M. (2008). *Introduction of time series analysis and forecasting.* New Jersey: John Wiley & Sons. Inc.

Nicita, A. (2013). Exchange rate, international trade and trade policies. *Policy issues in international trade and commodities.* New York and Geneva: United Nations.

Nikolsko-Rzhevesky, A., & Prodan, R. (2014). Markov switching and exchange rate predictablity. *International journal of forecasting* , 353-365.

Rahman, M., & Akhter, K. (2016). Addressing the trade facilication deficit: Raising export competitiveness in the Indian market. *Essays in honor of professor Alamgir Muhammad Serajuddin* . Chittagong, Bangladesh: University press limited.

Rogoff, K. S. (2008). The continuing puzzle of short-horizon exchange rate forecasting. *NBER working paper series* .

Rogoff, K. (1996). The purchasing power parity puzzle. *Journal of economic literature* , 647-68.

Rossi, B. (2013). Exchange rate predictability. *Journal of economic literature* , 1063-1119.

Sinha, P., & kohli, D. (2013). Modeling exchange rate dynamics in India using stock market indices and macroeconomic variables. *MPRA paper 45816* .

Taylor, A. M., & Taylor, M. P. (2004). The purchasing power parity debate. *Journal of ecoomic perspectives* , 135-158.

Taylor, M. P., & Sarno, L. (2001). Nonlinear mean reversion in real exchange rates: towards a solution to the purchasing power parity puzzles. *International economic review* , 1015-1042.

Tong, H. (1983). *Threshold models in non-linear time series analysis.* New York: Springer New York.

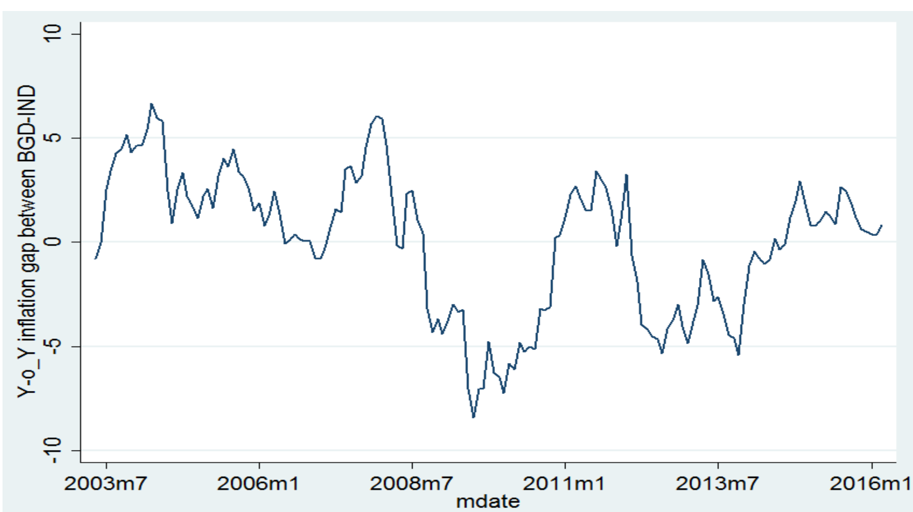
Tong, H. (2010). *Threshold models in time series analysis-30 years on.* The University of Hong Kong.

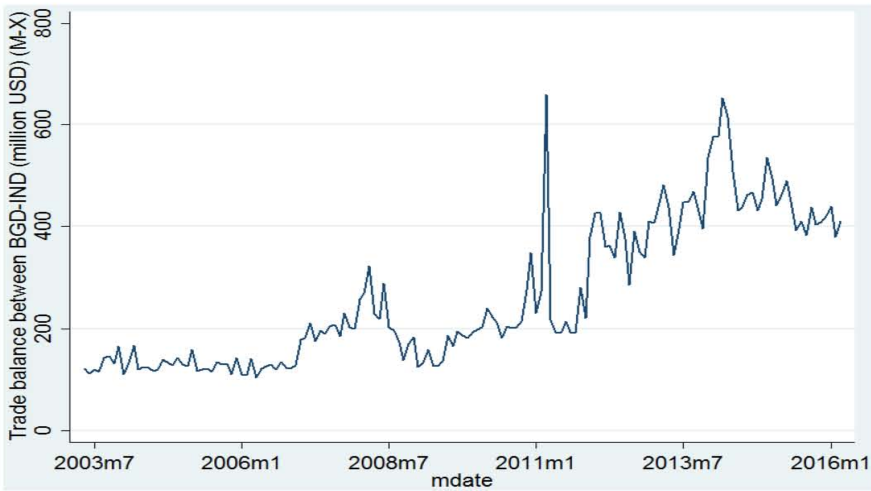
Woodall, P. (1986, September 6). On the hamburger standard. *The economist* .

Wu, J.-T. (2015). Markov regimes switching with monetary based exchange rate model. *Asia Pacific management review* , 79-89.

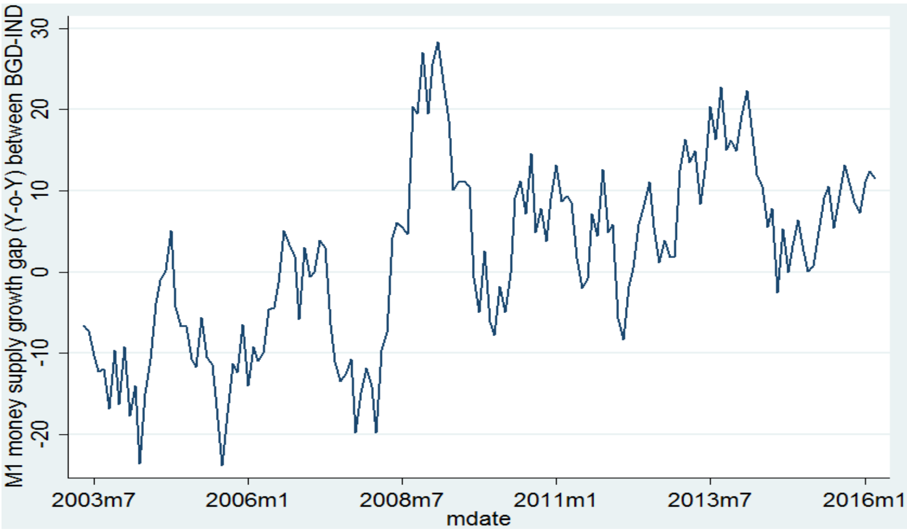
9. Appendix

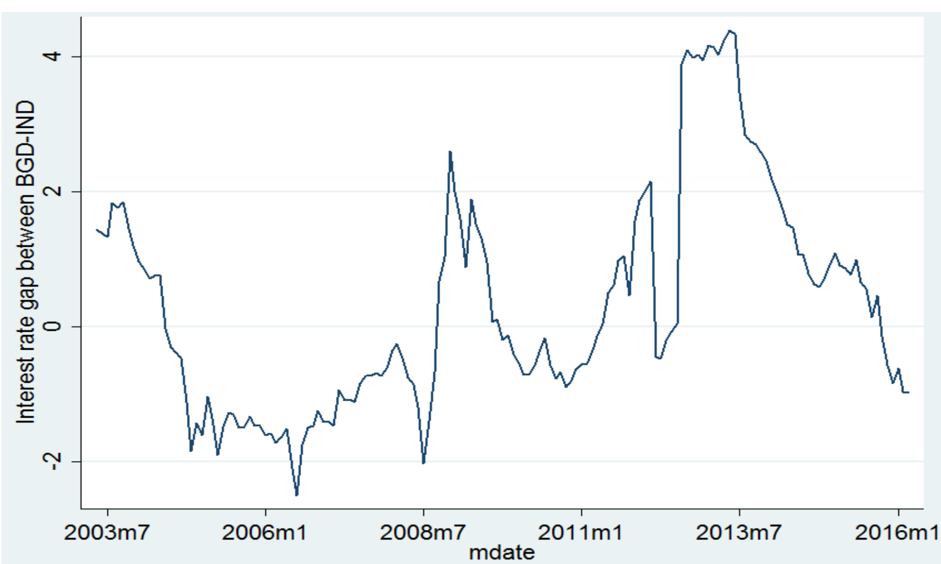
Appendix A: Figures

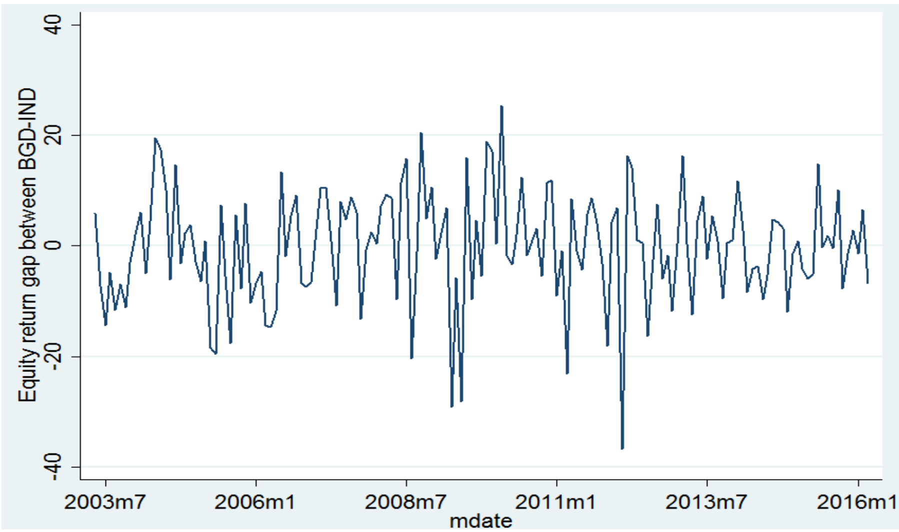



Trade balance between BGD-IND (million USD) (M-X)
0
200
400
600
800
2003m7
2006m1
2008m7
2011m1
2013m7
2016m1
mdate


Level gap in M2 money supply between BGD-IND
-2.2
-2
-1.8
-1.6
-1.4
-1.2
2003m7
2006m1
2008m7
2011m1
2013m7
2016m1
mdate


M1 money supply growth gap (Y-o-Y) between BGD-IND
30
20
10
0
-10
-20
2003m7
2006m1
2008m7
2011m1
2013m7
2016m1
mdate


Interest rate gap between BGD-IND
4
2
0
-2
2003m7
2006m1
2008m7
2011m1
2013m7
2016m1
mdate


Equity return gap between BGD-IND
40
20
0
-20
-40
2003m7
2006m1
2008m7
2011m1
2013m7
2016m1
mdate

Note:

Appendix C: STATA output for Markov switching parameter estimation

Engel & Hamilton model

```
Markov-switching dynamic regression

Sample:  2003m5 -  2016m3                        No. of obs       =        155
Number of states =     2                         AIC              =    -2.1358
Unconditional probabilities: transition          HQIC             =    -2.0879
                                                 SBIC             =    -2.0180
Log likelihood = 171.52296
```

tk_rup	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
State1						
_cons	1.258806	.0064868	194.06	0.000	1.246092	1.27152
State2						
_cons	1.532692	.0096995	158.02	0.000	1.513681	1.551703
sigma1	.0460465	.0046736			.03774	.0561813
sigma2	.0968378	.0068605			.0842833	.1112623
p11	.9890829	.012368			.9056365	.9988321
p21	.0140437	.0110029			.0029918	.0633294

. estat transition

Number of obs = 155

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9890829	.012368	.9056365	.9988321
p12	.0109171	.012368	.0011679	.0943635
p21	.0140437	.0110029	.0029918	.0633294
p22	.9859563	.0110029	.9366706	.9970082

. estat duration

Number of obs = 155

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	91.59968	103.774	10.59732	856.2701
State2	71.20617	55.78823	15.79045	334.2493

Purchasing power parity (PPP) model

```
Markov-switching dynamic regression

Sample:  2003m5 -  2016m3                         No. of obs       =       155
Number of states =    2                           AIC              =   -2.3504
Unconditional probabilities: transition           HQIC             =   -2.2946
                                                  SBIC             =   -2.2129

Log likelihood = 189.15504
```

tk_rup	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
tk_rup						
inf_gap_yoy	.010774	.0018021	5.98	0.000	.0072419	.0143061
State1						
_cons	1.245884	.0069184	180.08	0.000	1.232325	1.259444
State2						
_cons	1.538989	.0083584	184.12	0.000	1.522607	1.555371
sigma1	.0457394	.0050697			.0368082	.0568376
sigma2	.0818662	.0060948			.0707514	.0947272
p11	.9890915	.0123567			.9057315	.9988327
p21	.0140357	.0109956			.0029905	.0632856

```
. estat transition

Number of obs = 155
```

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9890915	.0123567	.9057315	.9988327
p12	.0109085	.0123567	.0011673	.0942685
p21	.0140357	.0109956	.0029905	.0632856
p22	.9859643	.0109956	.9367144	.9970095

```
. estat duration

Number of obs = 155
```

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	91.67192	103.8422	10.608	856.6826
State2	71.24703	55.81507	15.80138	334.3908

a)

Uncovered interest rate parity (UIP) model

```
Markov-switching dynamic regression

Sample:  2003m5 -  2016m3                        No. of obs      =        155
Number of states =     2                         AIC             =    -2.1275
Unconditional probabilities: transition          HQIC            =    -2.0717
                                                 SBIC            =    -1.9901
Log likelihood = 171.88142
```

tk_rup	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
tk_rup						
int_gap	-.0042281	.0050608	-0.84	0.403	-.0141471	.0056909
State1						
_cons	1.26301	.0082979	152.21	0.000	1.246746	1.279274
State2						
_cons	1.532863	.0095493	160.52	0.000	1.514146	1.551579
sigma1	.047615	.0052004			.0384395	.0589806
sigma2	.0950885	.006976			.0823532	.1097932
p11	.9891013	.0123427			.9058559	.9988331
p21	.014058	.0110164			.0029938	.0634109

. estat transition

Number of obs = 155

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9891013	.0123427	.9058559	.9988331
p12	.0108987	.0123427	.0011669	.0941441
p21	.014058	.0110164	.0029938	.0634109
p22	.985942	.0110164	.9365891	.9970062

. estat duration

Number of obs = 155

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	91.75436	103.9111	10.62202	856.9903
State2	71.13377	55.74291	15.77016	334.0192

Purchasing power parity and Uncovered interest rate parity (PPP+UIP)

```
Markov-switching dynamic regression

Sample:  2003m5 -  2016m3                          No. of obs        =        155
Number of states =    2                            AIC               =    -2.3409
Unconditional probabilities: transition            HQIC              =    -2.2771
                                                   SBIC              =    -2.1838
Log likelihood = 189.41861
```

tk_rup	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
tk_rup						
inf_gap_yoy	.0108129	.001809	5.98	0.000	.0072674	.0143584
eqret_gap	.0004099	.0005643	0.73	0.468	-.0006961	.001516
State1						
_cons	1.246185	.0069479	179.36	0.000	1.232567	1.259803
State2						
_cons	1.53943	.0083215	184.99	0.000	1.523121	1.55574
sigma1	.0461885	.0051188			.0371706	.0573941
sigma2	.0812723	.0061021			.0701508	.0941569
p11	.9891121	.0123282			.9059777	.9988338
p21	.0140514	.0110103			.0029928	.0633744

. estat transition

Number of obs = 155

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9891121	.0123282	.9059777	.9988338
p12	.0108879	.0123282	.0011662	.0940223
p21	.0140514	.0110103	.0029928	.0633744
p22	.9859486	.0110103	.9366256	.9970072

. estat duration

Number of obs = 155

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	91.84491	103.9948	10.63578	857.4742
State2	71.16754	55.76496	15.77924	334.135

Monetary exchange rate model-1

```
Markov-switching dynamic regression

Sample:  2003m5 -  2016m3                  No. of obs      =        155
Number of states =     2                   AIC             =    -3.5320
Unconditional probabilities: transition    HQIC            =    -3.4602
                                           SBIC            =    -3.3553
Log likelihood = 282.73015
```

ltk_rup	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
ltk_rup						
inf_gap_yoy	.000196	.0018553	0.11	0.916	-.0034404	.0038324
int_gap	.016416	.0026163	6.27	0.000	.0112882	.0215438
m2_lvgap	-.4675553	.0147542	-31.69	0.000	-.4964731	-.4386375
State1						
_cons	-.5415086	.0276969	-19.55	0.000	-.5957934	-.4872237
State2						
_cons	-.4216291	.0253656	-16.62	0.000	-.4713447	-.3719135
sigma1	.0410138	.008036			.0279351	.0602158
sigma2	.0348276	.0027619			.0298141	.0406842
p11	.9646236	.0287768			.8392827	.9930254
p21	.0146842	.0106378			.0035149	.0592367

. estat transition

Number of obs = 155

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9646236	.0287768	.8392827	.9930254
p12	.0353764	.0287768	.0069746	.1607173
p21	.0146842	.0106378	.0035149	.0592367
p22	.9853158	.0106378	.9407633	.9964851

. estat duration

Number of obs = 155

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	28.26743	22.994	6.222104	143.378
State2	68.10055	49.33497	16.88141	284.5065

Monetary exchange rate model-2

Markov-switching dynamic regression

Sample: 2003m5 - 2016m3	No. of obs	=	155
Number of states = 2	AIC	=	-2.5412
Unconditional probabilities: transition	HQIC	=	-2.4694
	SBIC	=	-2.3645

Log likelihood = 205.94341

tk_rup	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
tk_rup						
inf_gap_yoy	.0123608	.0017638	7.01	0.000	.0089039	.0158178
int_gap	-.0057769	.0038223	-1.51	0.131	-.0132685	.0017146
m1g_gap_yoy	-.0040933	.0005378	-7.61	0.000	-.0051474	-.0030393
State1						
_cons	1.284349	.0104563	122.83	0.000	1.263855	1.304843
State2						
_cons	1.572975	.0057291	274.56	0.000	1.561747	1.584204
sigma1	.0805155	.0079395			.0663658	.097682
sigma2	.0482362	.004258			.0405727	.0573471
p11	.9905483	.0103733			.9227614	.9989135
p21	.0156448	.0124896			.003232	.0722726

. estat transition

Number of obs = 155

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9905483	.0103733	.9227614	.9989135
p12	.0094517	.0103733	.0010865	.0772386
p21	.0156448	.0124896	.003232	.0722726
p22	.9843552	.0124896	.9277274	.996768

. estat duration

Number of obs = 155

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	105.8015	116.1178	12.94689	920.3488
State2	63.91904	51.02815	13.8365	309.4022

Monetary exchange rate+ Trade balance model

```
Markov-switching dynamic regression

Sample:  2003m5 -  2016m3                        No. of obs        =        155
Number of states =     2                         AIC               =    -2.5896
Unconditional probabilities: transition          HQIC              =    -2.5098
                                                 SBIC              =    -2.3932

Log likelihood = 210.69118
```

tk_rup	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
tk_rup						
inf_gap_yoy	.0120697	.0016636	7.26	0.000	.0088091	.0153302
int_gap	-.0097137	.0036414	-2.67	0.008	-.0168508	-.0025766
m1g_gap_yoy	-.0043527	.0005147	-8.46	0.000	-.0053614	-.0033439
tb	.0001342	.0000428	3.14	0.002	.0000503	.000218
State1						
_cons	1.246392	.0159319	78.23	0.000	1.215166	1.277618
State2						
_cons	1.542333	.0112077	137.61	0.000	1.520366	1.5643
sigma1	.0796684	.0075596			.0661481	.0959523
sigma2	.0459798	.0038831			.0389656	.0542567
p11	.9905427	.0103809			.9226966	.9989131
p21	.0156302	.0124758			.0032299	.0721891

. estat transition

Number of obs = 155

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9905427	.0103809	.9226966	.9989131
p12	.0094573	.0103809	.0010869	.0773034
p21	.0156302	.0124758	.0032299	.0721891
p22	.9843698	.0124758	.9278109	.9967701

. estat duration

Number of obs = 155

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	105.7379	116.0634	12.93604	920.0676
State2	63.97875	51.06703	13.8525	309.6032

Appendix D: STATA output for the forecasting exercise