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SPEED CONTROL OF DC MOTOR USING CONTROL SYSTEM ANALYSIS

By

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bstract

C motor speed controls are fundamental in vehicles in general and robotics in particular. This udy presents a mathematical model for correlating the interactions of some DC motor control arameters such as duty cycle, terminal voltage, frequency and load on some responses such as tput current, voltage and speed by means of response surface methodology. The significance f the mathematical model developed was ascertained using regression analysis method. The sults obtained show that the mathematical models are useful not only for predicting optimum C motor parameters for achieving the desired quality but for speed and position optimization. sing the optimal combination of these parameters is useful in minimizing the power onsumption and realization of the optimal speed and invariably position control of DC motor perations

his paper describes the MATLAB realization of the DC motor speed control methods, namely eld resistance, armature voltage and armature resistance control methods, and feedback control ystem for DC motor drives.

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Authorization page

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1. INTRODUCTION

DC motor and DC motor drives are very essential in the field of electrical as well as electronic devices. DC motors are commonly used for using its rotation and rotational speed for various applications. But controlling its rotational position and rotational speed is a task which is essential to be done while using it. This project is based on developing different scheme for controlling the speed of a DC motor mainly armature controlled DC motor.

DC motor characteristics are very important to study and analyze because it reveals the factors that controls the DC motor speed. First part of the paper is analyzing the theoretical modeling of a DC motor and using the equation and theoretical scheme we have find out the controlling parameter.

Latter part is about the control system analysis of DC motor which is the most essential part for serving our purpose which is to get a constant speed for whatever the condition is. More specifically any change in disturbance load, friction can not change the operating speed of the motor. To do so controlling parameter is not enough. We have to design close loop system using different scheme and then chose the most effective one to finalize our scheme.

For developing a close loop system first of all we used Root-Locus method. But depending on different limitation and problem we reject the process and developed an optimal control system. Then we finalize our scheme using LQR control.

2. DC MOTOR

2.1. Introduction

Electrical Machines is nowadays an essential part of our life. The usages of electrical machines are very extensive. But controlling the machines has become a very challenging factors rather than building kind of prototypes. There are many types of controlling procedure or scheme available for the purpose, but paper is mainly based on control system analysis. To use control system analysis approach to control the speed, a vital parameter of, of a DC Motor.

DC Motor speed control is a vital task to be done where speed of these machines is used. For instance, electrical vehicle, robotic motoring etc. Prior to access into the control analysis it would be used to analyze the theory behind the DC Motor.

This chapter is mainly based on theoretical overview of Dc Motor. Deferent equations derived from the equivalent of DC Motor are analyzed here in this section. Classification of DC Motor and different characteristics of each of the DC Motors are the main points which is to be discussed.

Final goal is to find out the factors that control the speed of a DC Motor which is the main purpose to be served. Choosing a factor which is the best to control the speed is the final approach of this section.

2.2. Principle of DC Motor

An electric motor is a machine which converts electric energy to mechanical energy. Its action is based on the principle that when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by **F = BIL** Newton. DC motors are like generators separately excited, shunt-wound or series-wound or compound wound. [1]

When the field magnets of the DC motors are excited and its armature conductors are supplied with current from the main supply, they experience a force tending to rotate the armature.

Armature conductors under N-pole are assumed to carry current downwards and those under S-poles to carry current upwards. The direction of the force is on each conductor can be found by applying Flemings' Left hand Rule. It will be seen that each conductor experiences a force F which tends to rotate the armature in anti-clockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

2.2.1. Equations of a DC Motor

The voltage V applied across the motor armature has to

Overcome the back e.m.f E_b

Supply the armature voltage drop $I_a R_a$

So, $V = E_b + I_a R_a$ **2.1**

This is known as voltage equation of a motor. Now multiplying both sides by I_a, the equation become

$V I_a = E_b I_a + I_a^2 R_a$ **2.2**

$V I_a$ = electrical input power to the armature

$E_b I_a$ = electrical equivalent of mechanical power developed in the armature

$I_a^2 R_a$ = Cu loss in the armature

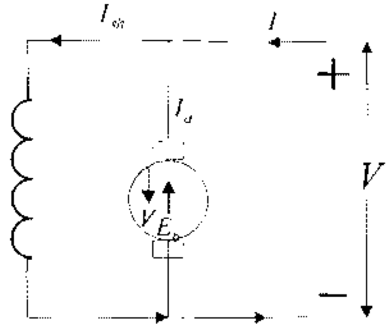

Figure 2.1: Equivalent circuit of a DC motor

So, without $I_a^2 R_a$ loss, the rest is converted into mechanical power within the armature.

It may also be noted that motor efficiency is given by the ratio of power developed by the armature to its input. It is obvious that the higher value of E_b as compared to V, higher the motor efficiency.

2.2.2. Significance of the Back E.M.F :

When the motor armature rotates, the conductor also rotates and hence the flux. In accordance with the locus of the electromagnetic induction e.m.f is induced in them whose direction is found by applying Flemings' Right hand Rule which is opposition to the apply voltage. For its opposite direction, it is referred to a counter e.m.f or back e.m.f. **[1]**

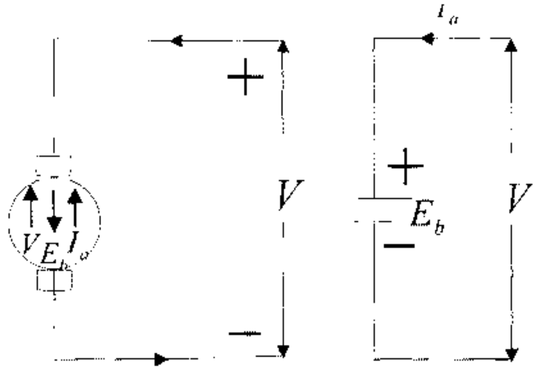

Figure 2.2: Equivalent circuit including back e.m.f

e supply V has to drive I_a against the opposition of E_b. The power required to overcome this osition is E_bI_b. In the case of cell, this power over an interval of time is converted in to mical energy, but in the case of motor, it is converted into mechanical energy.

ill be seen that **I_a = (net voltage / resistance) = (V-V_b/R_a)**; where R_a is the armature stance. And

= V_b = **ΦZN** volt 2.3

ere N is in r.p.s (rotation per second).

ed is high, E_b is high, hence armature current I_a according to above equation is small. If is less, E_b is less, hence more current flows which develop more torque. So, it is found E_b

acts like a governor i.e it makes a motor self regulator so that it draws as much current as is just necessary.

2.2.3. Condition for maximum Power

The gross mechanical power developed by the motor is

$P_m = V\ I_a - I_a^2\ R_a$ 2.4

Differentiating both sides with respect to I_a and equating the result is zero, it gives

$(d\ P_m / d\ I_a) = V - 2\ I_a\ R_a = 0$

$V = 2\ I_a\ R_a$

$V/2 = I_a\ R_a$ 2.5

As,

$V = E_b + I_a R_a$ 2.6

$V = E_b + V/2$

$V/2 = E_b$ 2.7

Thus gross mechanical power developed by a motor is maximum when back e.m.f is equal to half of the applied voltage. This condition is however, not realized in practice because in that case current would be much beyond the normal current to the motor. Moreover, half of the input would be wasted in the form of heat and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be below **50%. [1]**

2.3. Torque Analysis

2.3.1. Torque

By the term torque it is meant by turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius of which this force acts. Consider, pulley of radius r meter acted upon by a circumferential force of F Newton which causes it to rotate at N r.p.s.

Torque, $\mathbf{T = F \times r}$ Newton-meter **2.8**

Work down by this force in one revolution = Force × Distance = $F \times 2\pi r$ joule

Power developed = $\mathbf{F \times 2\pi rN}$ **watt**

$= \mathbf{(F \times r) \times 2\pi N}$ **watt** **2.9**

Now, $\mathbf{2\pi N = \omega}$ is radian / second angular velocity

Power developed, $\mathbf{P = T \times \omega}$ **watt** **2.10**

If N is r.p.m (rotation per minute), then

$\mathbf{P = T \times (2\pi N/60)}$ **watt**

$\mathbf{P = TN \times (2\pi/60)}$ **watt**

$\mathbf{P = TN / 9.55}$ **watt** **2.11**

2.3.2. Armature Torque of a Motor

Let T_a be the torque developed by the armature of a motor running at N r.p.s. Then power developed

$= T_a \times \mathbf{2\pi N}$ **watt** **2.12**

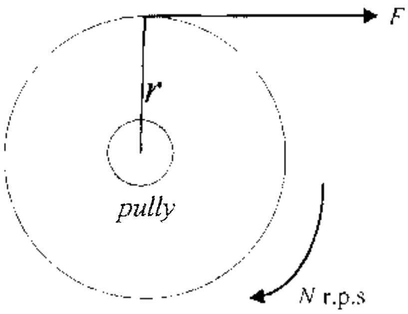

Figure 2.3: Torque of a rotating object

It is known that the electrical power converted into mechanical power in the armature is $\mathbf{E_b I_a}$ **watt 2.13**

From equation **2.12 & 2.13**, it arrives

$\mathbf{T_a \times 2\pi N = E_b I_a}$

$\mathbf{T_a \times 2\pi N = \Phi Z N \times I_a}$

$\mathbf{T_a = (1/2\pi) \times \Phi Z I_a}$

$\mathbf{T_a = 0.159\ \Phi Z I_a}$ **N-m 2.14**

From the above equation,

$\mathbf{T_a \alpha \Phi I_a}$ **.. 2.15**

In case of series motor, **Φ** is directly proportional to $\mathbf{I_a}$ because field windings carry full armature current. So

$\mathbf{T_a \alpha I_a^2}$ **... 2.16**

In case of shunt motor **Φ** is constant, hence

$\mathbf{T_a \alpha I_a}$ **... 2.17**

2.3.3. Shaft Torque

The whole of the armature torque is not available for doing useful work because a certain amount of it is required for supplying iron and friction losses in the motor. The torque which is available for doing useful work is known as shaft torque $\mathbf{T_{sh}}$. The motor output is given by

$T_{sh} \times 2\pi N$ **watt 2.18**

where N is r.p.s

T_{sh} **= (Output / 2πN) N-m-N in r.p.s**

= Output in watts / (2π/60) N-m-N in r.p.m

= (60/2π) × (Output / N)

T_{sh} **= 9.55 × (Output / N) 2.19**

The difference (T_s-T_{sh}) is known as lost torque and is due to iron and friction losses of the motor.

2.3.4. Torque and Speed of a DC Motor

The equation of speed motor is

$N = K \times (V - I_aR_a)/\Phi = K\, E_b/\Phi$ 2.20

And, **$T_a \;\alpha\; \Phi I_a$ 2.21**

It is seen from above that increase in flux would decrease the speed but increase the armature torque. If torque increases, motor speed must increase rather than decrease.

Suppose that the flux of a motor is decreased by decreasing field current. Then, the following sequences of events take place:

Back e.m.f **$E_b = N\,\Phi/K$** drops instantly due to decrease in **E_b**, **I_a** is increased because I_a = **$(V-E_b)/R_a$** . Moreover, a small reduction in flux produces a large increase in armature current.

Hence, the equation **$T_a \;\alpha\; \Phi I_a$**, a small decrease in **Φ** is more than counterbalanced by a large increase in **I_a** with the result that there is no increase in **T_a**.

This increase in T_a produces the increase in motor speed.

It is observed from the above that, with the applied voltage V held constant motor speed varies inversely as the flux. However, it is possible to increase flux and at the same time increase the speed provided **I_a** is held constant as is actually done in a DC servomotor.

2.4. Classification and Characteristics

DC motor's classification and characteristics are of great importance while controlling the speed of DC motors. Because different types of DC motors refers to different controlling scheme. So we have to have an clear idea over the characteristics of different types of DC motor. Classifications and characteristics are given in the next sub-section. **[1]**

2.4.1. Classification

DC Motor can be classified as stated below:

1. Series wound DC Motor (self-excited).
2. Shunt wound DC Motor (self-excited).
3. Separately excited DC Motor

We in this project is trying to control the separately excited DC Motor. So separately excited DC Motor and its characteristics are of great importance for our purpose.

2.4.2. Characteristics

The characteristics curves of a motor are those curves which show relationship between the following equation:-

Torque and armature current i.e. T_a/I_a characteristics.

Speed and armature current i.e. N/I_a characteristics.

Speed and torque i.e. N/T_a characteristics.

The following two equations are really important-----

$T_a \propto \Phi I_a$ and $N = K E_b/\Phi$ or $N \propto E_b/\Phi$

2.4.2.1. Characteristics of a series motor

T_a/I_a characteristics:

... observed that $T_a \propto \Phi I_a$. In this case, as field windings also carry the armature current,

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$\Phi \alpha I_a$ up to the part of magnetic saturation. Hence, before saturation,

$T_a \alpha \Phi I_a$ and $T_a \alpha I_a^2$

At light load as, I_a and Φ is small. But as I_a increases, T_a increases as the square of the current. Hence, T_a / I_a curve is a parabola. After saturation Φ is almost independent of I_a hence $T_a \alpha I_a$ only. So the characteristic becomes a straight line. The shaft torque T_{sh} is less than armature torque due to stay losses. It is shown in dotted line. So, it can be concluded that on heavy loads, a series motor exerts a torque proportional to the square of armature current. Hence, in cases where huge starting torque is required for accelerating heavy masses quickly as in hoists and electric trains etc. , series motors are used.

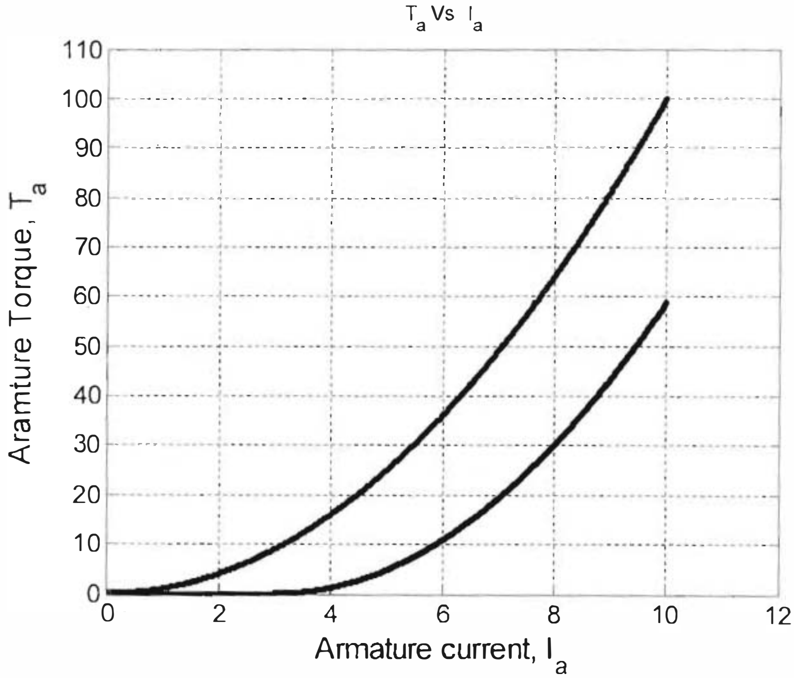

Figure 2.4: T_a V_s I_a Characteristics

N I_a characteristics:

We know that, $N \alpha E_b / \Phi$. With increased I_a, Φ also increased. Hence, speed varies inversely as armature current as shown in figure.

When load is heavy, I_a is large. Hence, speed is low. But when load current and I_a falls to a small value. speed become dangerously high. Hence, a series motor never be started without some mechanical load on it otherwise it may be develop excessive speed and get damaged due to heavy centrifugal forces so produced. It should be noted that series motor is a variable speed motor.

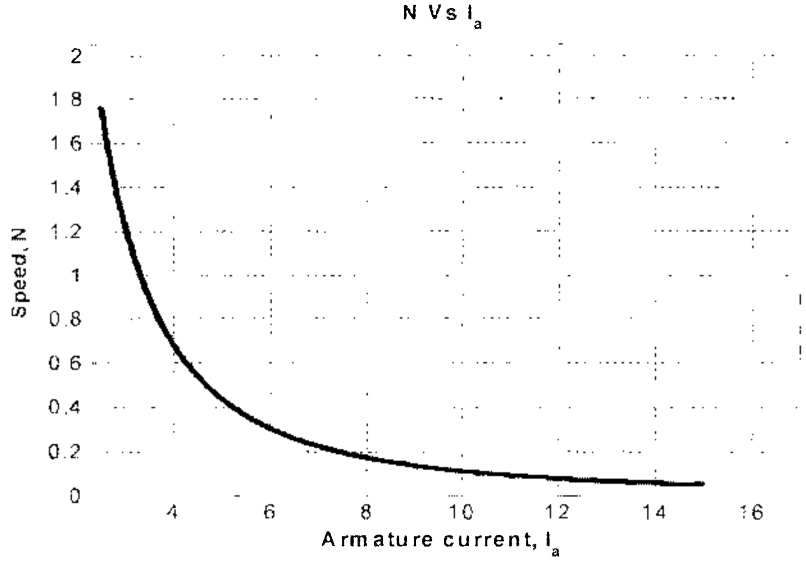

Figure 2.5: I_a V_s N Characteristics of a series motor

N. T_a characteristics:

It is found that when speed is high, torque is low and vice-versa.

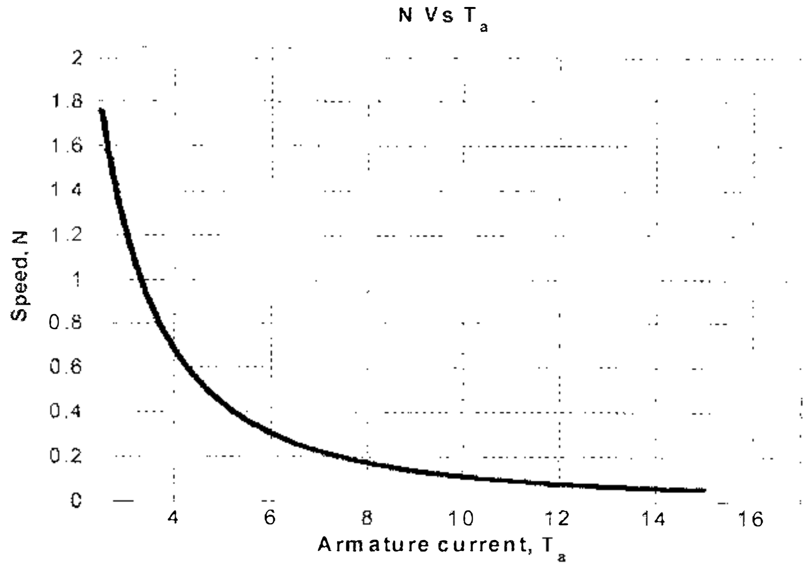

Figure 2.6: T_a V_s N Characteristics of a series motor

2.4.2.2. Characteristics of a shunt motor

T_a/ I_a characteristics:

For shunt motor, **Φ** is to be practically constant and we find that **T_a α I_a** . For a heavy starting load will need a heavy starting current, shunt motor should never be started on heavy load.

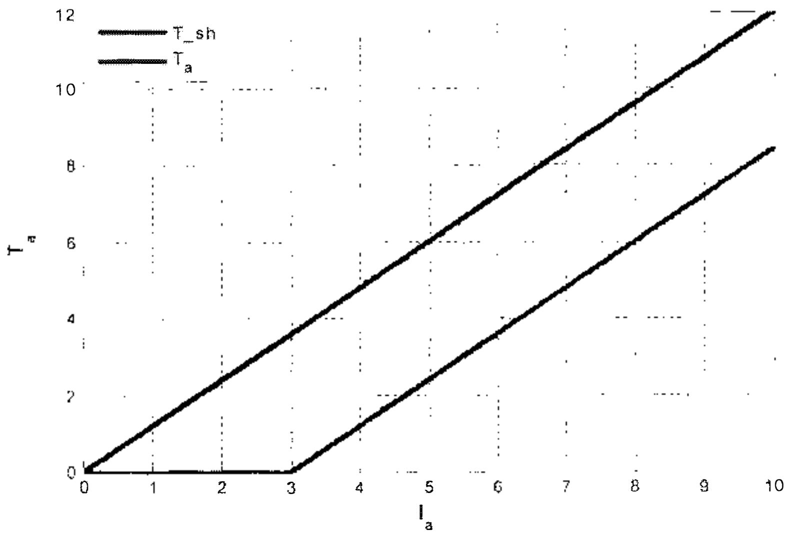

Figure 2.7: I_a V_s T_a Characteristics of a shunt motor

N/ T_a characteristics:

If **Φ** is assumed constant, then **N α E_b**. As **E_b** is also practically constant, speed is also constant.

But both E_b and **Φ** decrease with increasing load. However, **E_b** decreases slightly more than **Φ** so that on whole, there is some decrease in speed. The drop varies from 5 to15% of full-load speed, being dependent on saturation, armature reaction and brush position. Hence, the actual speed curve is slightly drooping as shown by the dotted line. But for a practical purpose, shunt motor is taken as a constant-speed motor. Because there is no appreciable change in the speed of shunt motor from no-load, it may be connected to load which are totally and suddenly thrown off without any fear of excessive speed resulting. Due to the constancy of their speed, shunt motors are suitable for driving shafting, machine tools, wood-working machines and for all other purposes where on approximately constant speed is required.

N/ I_a characteristics:

As the shunt motor is as constant speed motor, so its angular velocity ω is a constant parameter whatever the armature current is. Therefore, armature current Vs angular velocity curve is straight line at least theoretically. But it is not the case in reality or practically. Practically angular velocity or speed decreases with the increase of armature current (or in other words increase in torque demand or load). But the rate of change is very slight as described in the following Figure 2.8.

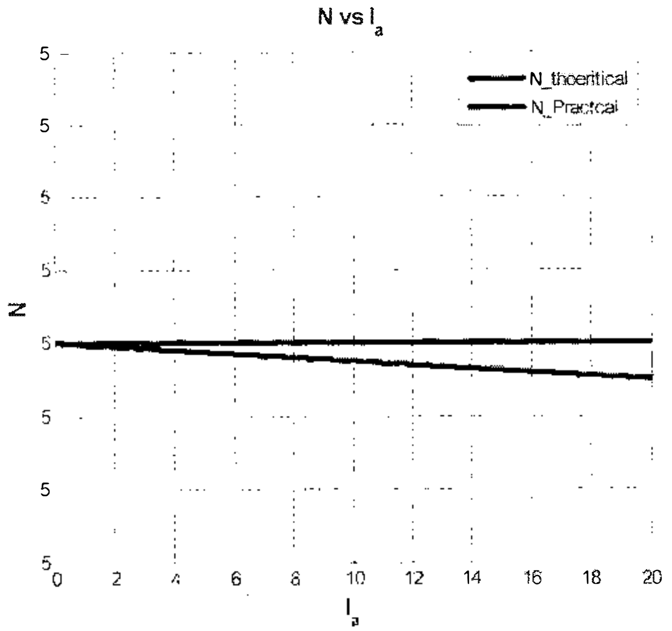

Figure 2.8: I_a V_s N Characteristics of a Shunt motor

2.4.2.3. Characteristics of a separately excited DC Motor

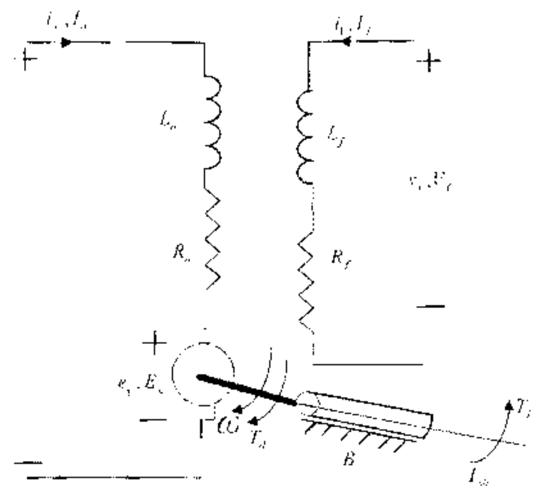

Figure 2.9: Equivalent circuit of separately excited DC Motor

...en a separately excited motor is excited by a field current of i_f and an armature current of $\mathbf{i_a}$, ... motor develops a back electromotive force and a torque to balance the load torque at a ...ticular speed. The field current i_f of a separately excited motor is independent of the armature current i_a of and any change in armature current has no effect on the field current. The field current is normally much less than the armature current. [2]

The instantaneous field current i_f is described as—

$V_f = R_f i_f + L_f (di_f/dt)$ 2.22

The instantaneous armature current can be found from

$V_a=R_a i_a+L_a(di_a/dt)+e_g$ 2.23

The motor back emf, which is known as speed voltage is expressed

$e_g=K_v\omega\, i_f$ 2.24

where ω=angular speed, K_v=Voltage constant;

The torque developed by the motor is

$T_d=K_f i_f i_a$; 2.25

The developed torque must be equal to load torque

$T_d=J(d\omega/dt)+B\omega+T_L$ 2.26

Where, B = friction constant.

Under steady-state conditions, the time derivatives in these equations are zero and the steady-state average quantities are

$V_f=R_f I_f$ 2.27

$E_g=K_v\omega I_f$ 2.28

$V_a=R_a I_a+E_g$ 2.29

$V_a=R_a I_a+ K_v\omega I_f$ 2.30

$T_d=K_f I_a I_f$ 2.31

$T_d=B\,\omega +T_L$ 2.32

The developed power is-

$P_d= T_d\,\omega$ 2.33

The relationship between the field current I_f and the back emf, E_g is nonlinear due to magnetic saturation. The speed of a separately excited motor can be found from

$\omega = (V_a - R_a I)/ K_v I_f$ 2.34

From equation 2.34, we can notice that the motor speed can be varied by

Controlling the armature voltage, V_a.

Controlling field current, I_f.

Controlling armature current, I_a.

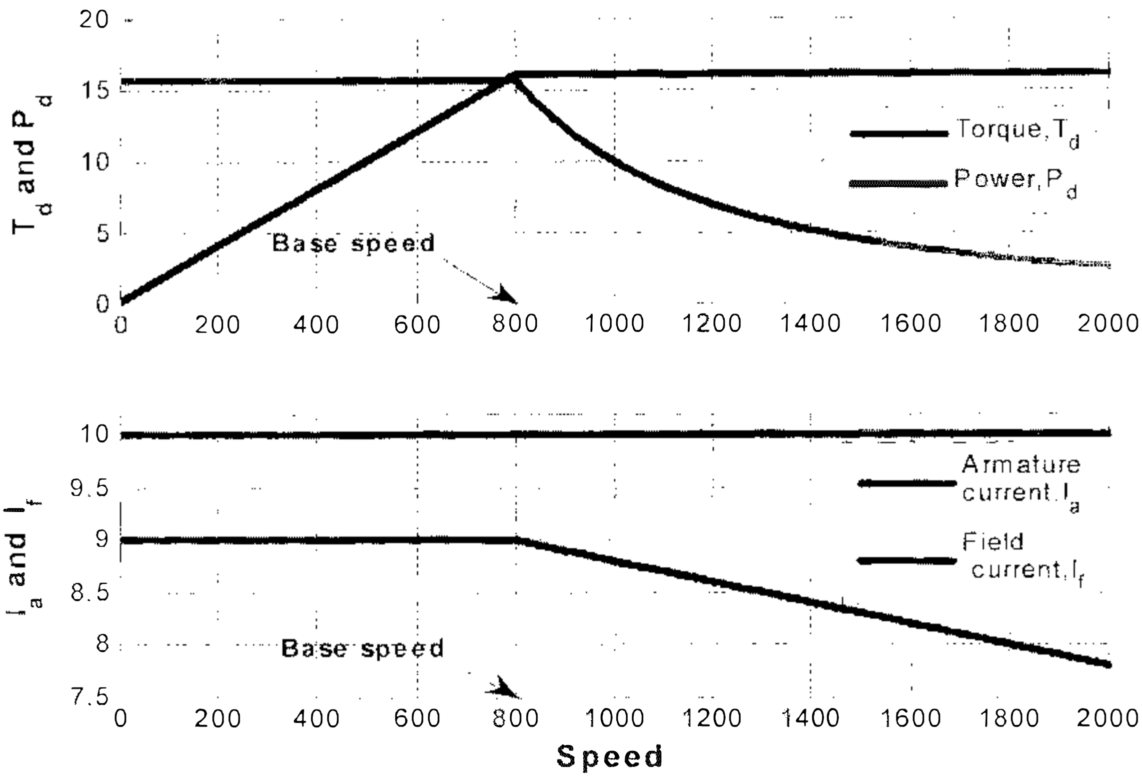

Figure 2.10: Characteristics of separately excited DC Motor

practice, for a speed less than the base speed, the armature current and field currents are ntained constant to meet the torque demand and the armature voltage, V_a is varied to control speed. For speed higher than the base speed, the armature is maintained at the rated value and current is varied to control the speed. However, the power developed by the motor (P_d =) remains constant. [2]

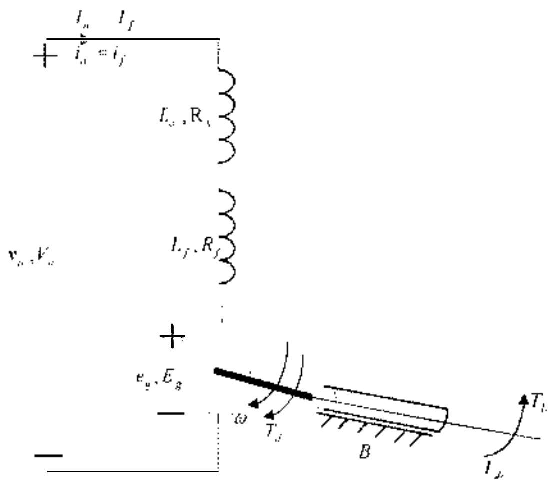

Figure 2.11: Equivalent circuit of series DC Motor

The field of a dc motor is connected in series with the armature circuit and this type of motor is called a series motor. The field circuit is designed to carry the armature current. The steady-state average quantities are,

$E_g = K_v \omega I_a$ **2.35**

$V_a = (Ra + R_f) Ia + E_g$

$= (Ra + R_f) Ia + K_v \omega I_f$ **2.36**

$\omega = (V_a - (Ra + R_f) I_a) / (K_v I_f)$ **2.37**

$T_d = B\,\omega + T_L$ **2.38**

From equation 2.37, the speed can be varied by controlling Armature voltage, V_a.
Armature current, which is a measure of the torque demand Equation 2.38 indicates that a series motor can provide a high torque specially at starting and form this reason this, series motor are commonly used in traction application.

For a speed up to the base speed, the armature voltage is varied and the torque is maintained constant. Once the rated armature voltage is applied, the speed-torque relationship follows the natural characteristics of the motor and the power remains constant. As the torque demand is reduced, the speed increases. At a very light load, the speed could be very high and it is not advisable to run a dc series motor without a load.

2.4.3. Factors Controlling Motor speed

The speed of a motor is given by the relation-

$N = K \times (V - I_a R_a) / \Phi$ r.p.s 2.39

Where, R_a- armature circuit resistance.

It is obvious that the speed can be controlled by varying-

1. Flux or pole **Φ** (Flux controlled)
2. Resistance **R_a** of armature circuit (Rheostatic control)
3. Applied voltage **V** (voltage control).

So, from the above discussion we can say that DC Motor speed can be varied by controlling flux or pole **Φ,** (equivalently by changing field current). We can also use armature resistance to control the DC motor speed. The most effective and easy way to controlling DC motor speed is the applied voltage.

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3. CONTROL SYSTEM ANALYSIS

3.1. Introduction

A system that maintains a relationship between the output and the reference input and using the difference as means of control is called feedback control system.

Feedback control systems are not limited to engineering but can be found in various non engineering fields as well. For instance, the human body is a highly advanced feedback control system. Both body temperature and blood pressure are kept constant by means of physiological feedback. In fact, feedback performs a vital function. It makes human body relatively intensive to external disturbances, thus enabling it to function properly in a changing environment.

Close-loop control system

Feedback control systems are often referred to as closed-loop control system. In a close-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal, is fed to the controller so as to reduce the error and bring the output of the system to a desired value. The term close-loop control system implies the use of feedback control action in order to reduce the system error. **[3]**

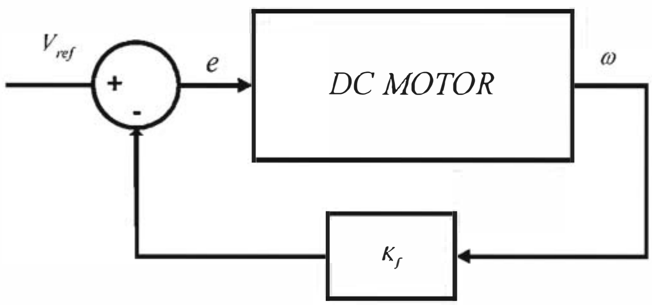

Figure 3.1: A simple Close-loop System

Open-loop control system

The system in which the output signal is generated in response to an input signal. For example, washing matching is a open-loop system. Soaking, washing and rinsing in the washer operate on the time basis. The machine does not measure the output signal i.e. the cleanliness of the clothes.

In any open-loop system the output is not compared with the reference input. In the presence of disturbance, an open-loop control system will not perform the desired task. Open-loop control system can be used only if the relationship between the input and output is known and there is no internal or external disturbance. It is important to note that any control system that operates on a time basis is open loop. [3]

Figure 3.2: A simple open-loop System

Close-loop versus open-loop control system

Closed-loop control system makes the system response relatively insensitive to external disturbances and internal variations in system parameters. But it is impossible in the open-loop case.

Open-loop control system is easier to build because system stability is not a major problem. On the other hand, stability is a major problem in the close-loop control system. Open-loop control system is used in those systems in which the inputs are known ahead of time and there are no disturbances. Close-loop control system is used in those systems in which unpredictable disturbances or unpredictable variations in system component are present.

The close-loop control system is generally higher in cost and power. To decrease the required power of a system, open-loop control may be used where applicable. [4]

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3.2. S-Domain modeling of armature controlled DC Motor

Transfer function and the block diagram of a armature controlled DC Motor is given below

$$\frac{\omega(s)}{V_a(s)} = \frac{K_m}{(R_a + L_a s)(Js+b) + K_b K_m} \quad \ldots\ldots 3.1$$

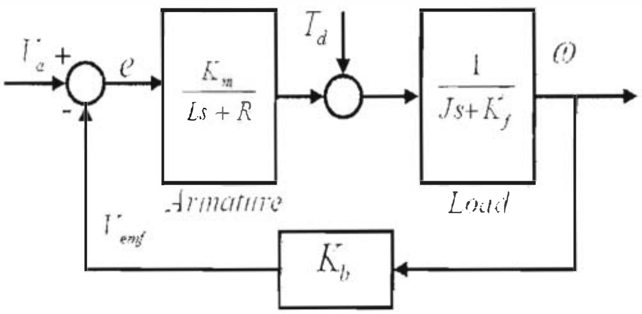

Figure 3.3: Block-Diagram of a Armature controlled DC motor

Step response of armature controlled DC motor is given below

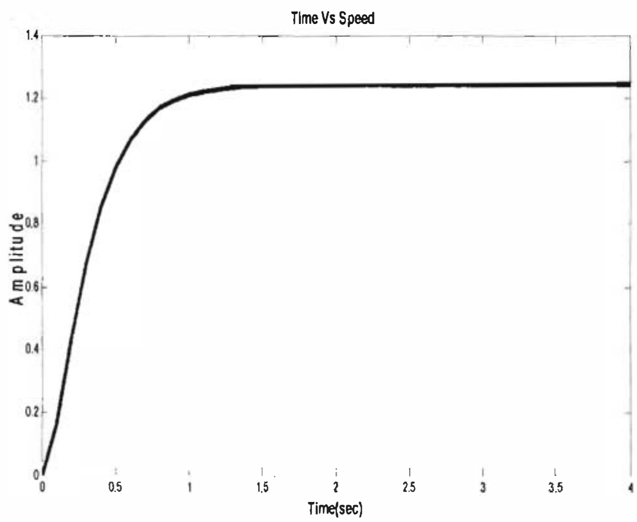

Figure 3.4: Step Response of armature controlled DC Motor

The effect of armature resistance on DC motor speed is given in Figure 3.5.

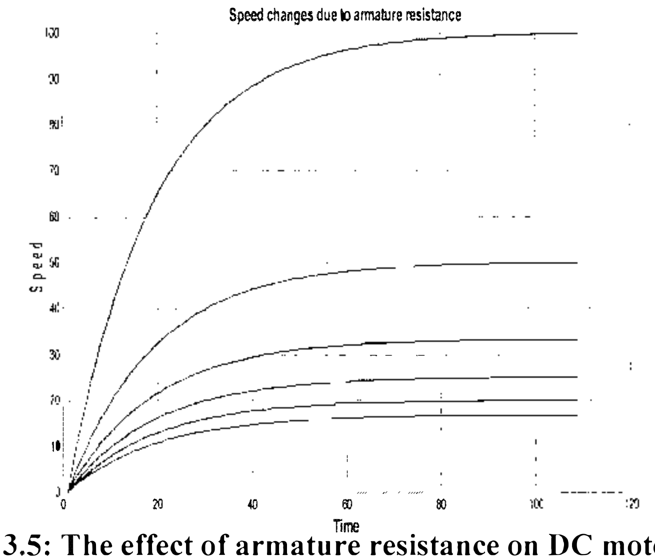

Figure 3.5: The effect of armature resistance on DC motor speed

From the above figure we can say that if the armature resistance increases then the speed of the DC motor decreases and vice-versa. But it is not the actual case. To observe the actual case it is recalled the speed equation of the DC motor.

$$N = K \times (V - I_a R_a) / \Phi = K E_b / \Phi$$

Initially if the armature resistance increases then the armature current decrease and magnetic flux decrease at the same time. But increase in armature current is greater than the corresponding flux. So, the nominator value increases and the denominator decrease and thus the speed increases. After a certain increasing value of armature resistance the speed is decreased for the increase of armature resistance. **[5]**

The effect of Voltage on speed

The speed and voltage have a proportionate relation. So, if the voltage increases the speed increase and vice-versa. This phenomenon is illustrated in the Figure 3.6. Here in this figure response is started after 35 second due to the input step signal that is started after 35 second.

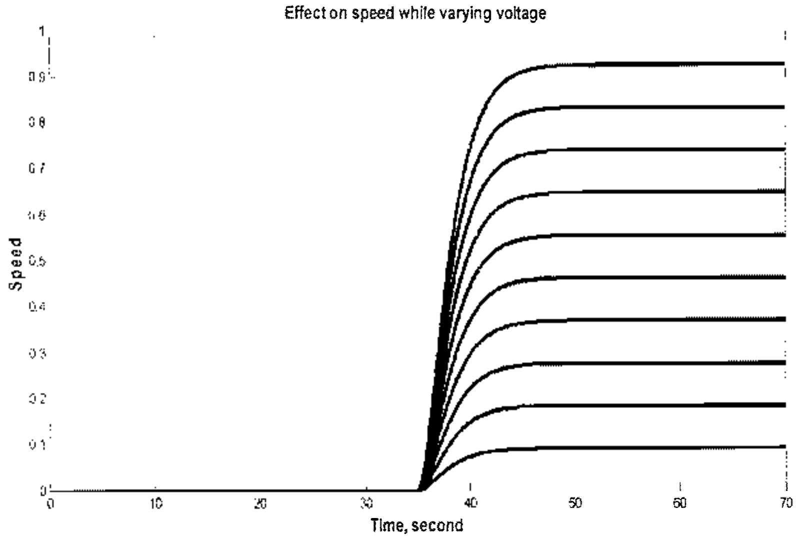

Figure 3.6: Effect of Voltage on Speed

Effect on step response taking La/Ra in consideration

If the value of **La/Ra** is large then the armature resistance has an effect on the speed of the DC Motor. But if the **La/Ra** is small then there are a little effect on the speed of the DC Motor. The effect on step response taking **La/Ra** in consideration is illustrated in Figure 3.7.

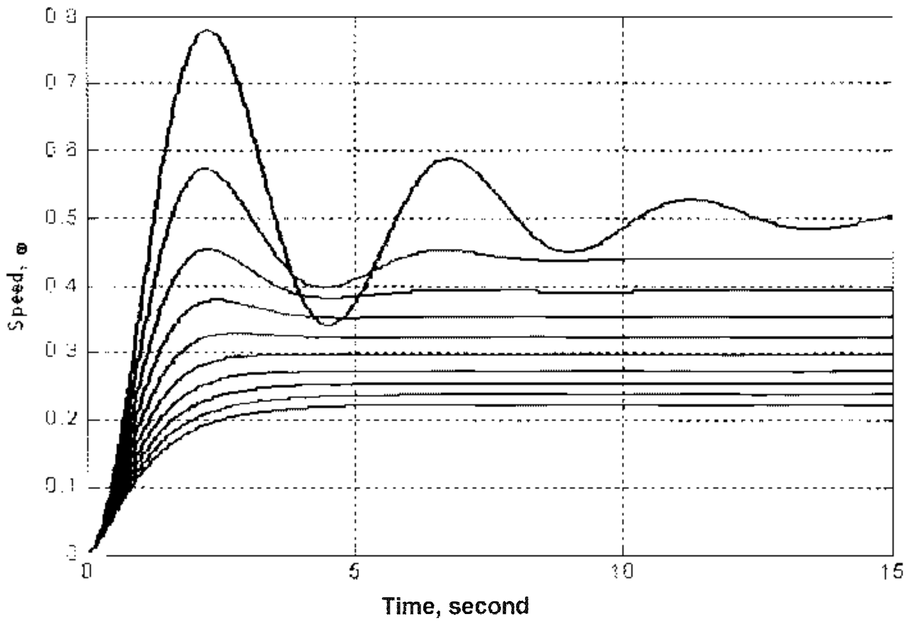

Figure 3.7: Effect on step response taking La/Ra in consideration

The above figure illustrates that if the value of armature inductance is very big comparing the armature resistance then initially there are rippling at the output and it takes some time to stable the system. Then it is called an under damped system because the damping coefficient is not too small which includes the armature resistance. If the value of armature resistance is increased then the damping coefficient is increased and therefore the system corresponds to a stable system. [5]

3.3. S-domain modeling of field controlled DC Motor

Transfer function and the block diagram of a armature controlled DC Motor is given below. [6]

$$\frac{\omega(s)}{V_f(s)} = \frac{K}{(R_f + L_f s)(Js + b)} \quad \ldots (3.2)$$

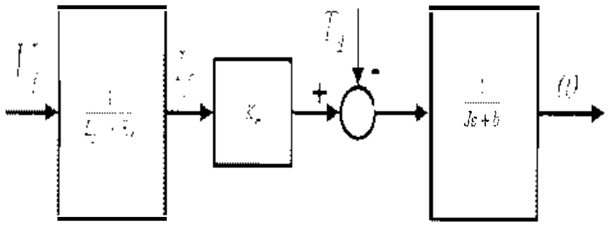

Figure 3.8: Block Diagram of Field Controlled DC motor

Step response of a field controlled DC Motor is given below

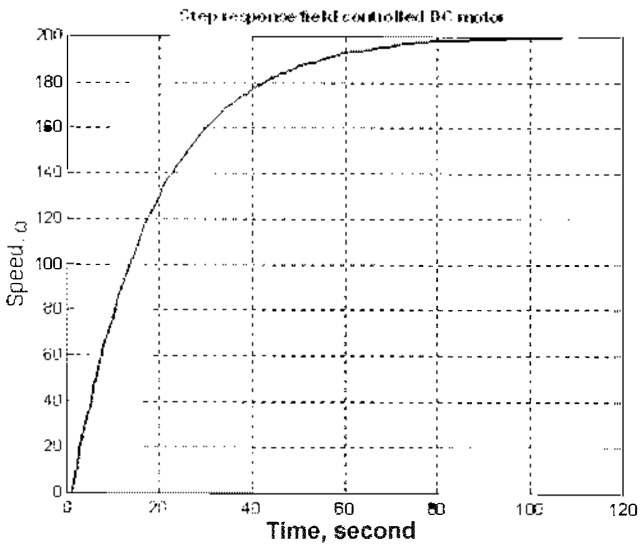

Figure 3.9: Step Response of field Controlled DC Motor

Effect of field resistance on step response

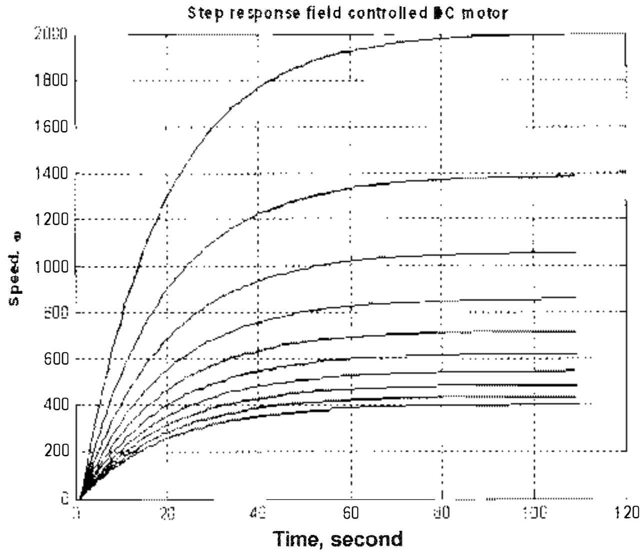

Figure 3.10: Effect of field resistance on step response

If the field resistance is increased then the speed of the DC Motor is decreased and vice – versa. It can be explained by the speed equation of the DC Motor. If the armature resistance is increases then the field current is decrease thus the field flux is decreased. So the speed of the DC motor is decreased and otherwise can be explained.

Feedback control of a DC motor with a feedback gain K_f

A simple circuit diagram of a feedback control of a system is given in the Figure 3.11. In the figure there is a feedback gain K_f. Feedback control reveals some importance why this feedback control is used to control some parameter.

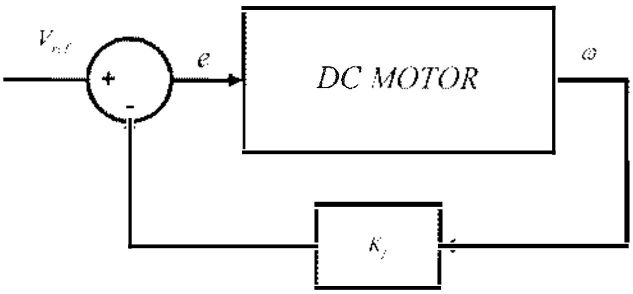

Figure 3.11: A feedback network with a proportionate feedback gain.

Figure 3.11 illustrates the comparison between a open-loop and a close loop system. The settling time of an open loop-system is bigger than that of a close-loop system. It means in an open-loop system it takes more time to settle the output, but a close-loop system takes less time to settle the output. In the following figure speed is negative. It is because the simulation is done taking voltage source zero and disturbance torque 0.1Nm as an input. This simulation is done only for observing and comparing the settling time of a open-loop and close-loop system. [6]

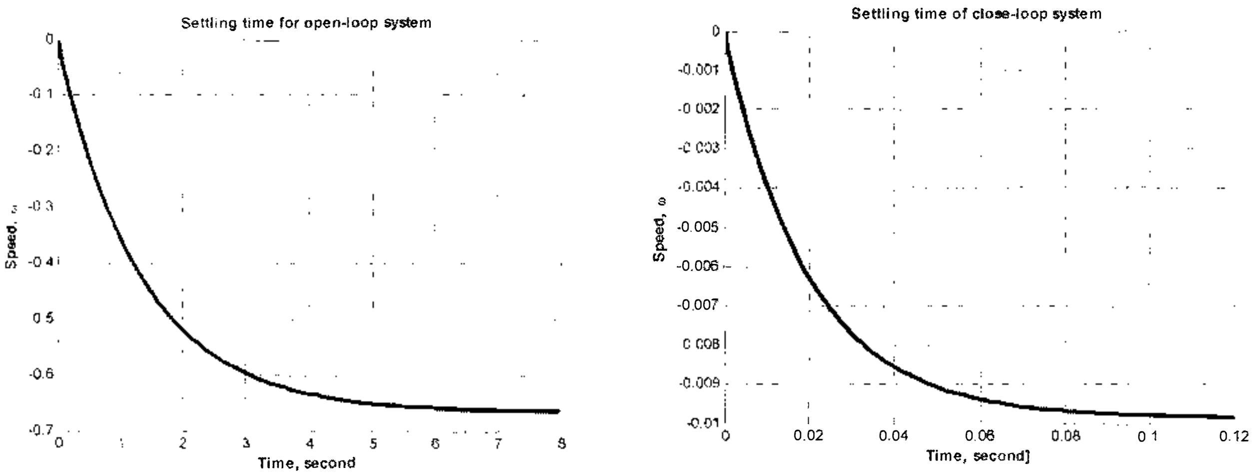

Figure 3.11: Settling time of open loop and Close loop system

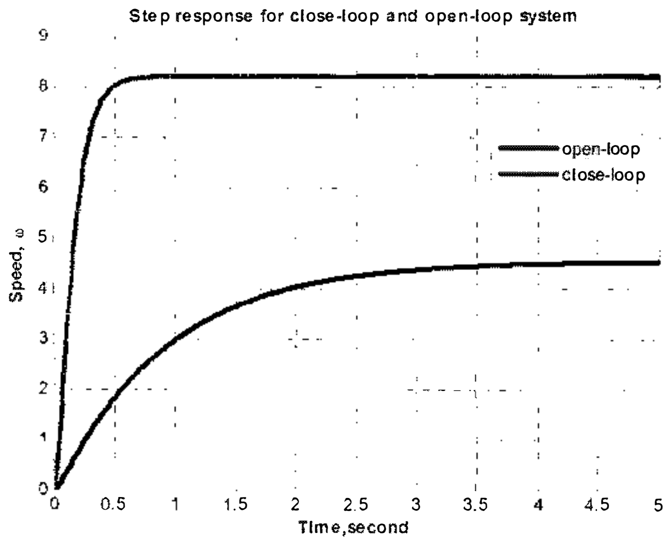

Figure 3.12: Step Response for an open-loop and Close loop System

Figure 3.12 also explained the comparison between an open-loop and a close-loop system. An open-loop system has a large settling time and less stable output value and vice-versa.

Effect of feedback gain on step response

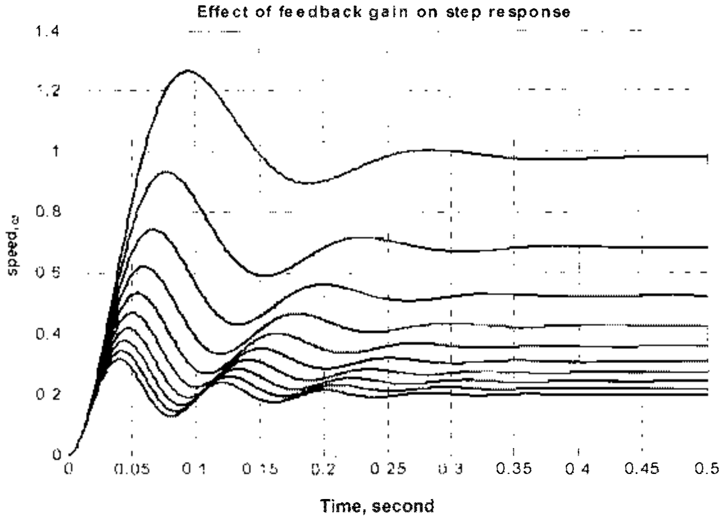

Figure 3.13: Effect of feedback gain on step response

Feedback gain of a close-loop system has a great effect on the response of a response. If the system feedback gain is kept increasing then the system keep generating more unstable output. It can be explained by the root-locus of the system. By this explanation, it can be said that if the gain value is increased the poles moves toward the right side of the s-plain and more rippling is introduced in the output. For a stable system its pole must be on the left side of the s-plain. So, if the value of feedback gain is increased, rippling is introduced in the output as the poles move toward the s-plain. [7]

3.4 State space modeling of a DC Motor

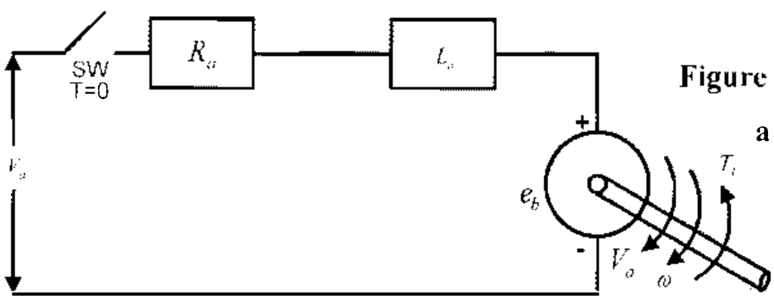

Figure 3.14: Equivalent circuit of a armature controlled DC Motor

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The motor torque **T** is related to the armature current, **i** , by a torque constant **K**;

$T(t) = Ki(t) - T_d(t)$ 3.3

Where, $\mathbf{T_d}$ is the disturbance torque.

Where, $\mathbf{T_d}$ is the disturbance torque.
The generated voltage, $\mathbf{e_b}$, is relative to angular velocity by;

$e_b = K\omega$ 3.4

From Figure 3.14 we can write the following equations based on the Newton's law combined with the Kirchoff's law

$$L\frac{d^2\omega(t)}{dt} - b\omega(t) = ki(t) - T_d(t) \text{ .... 3.5}$$

$$L\frac{di(t)}{dt} + Ri(t) = V_a(t) - K\omega(t) \text{ .... 3.6}$$

From the above two equations we can find out the state-space matrix as follows

$$\frac{d}{dt}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix} = \begin{bmatrix}\frac{-b}{j} & \frac{K_m}{j}\\ \frac{-K_b}{L} & \frac{-R}{L}\end{bmatrix}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix} + \begin{bmatrix}-1 & \\ & 1\end{bmatrix}\begin{bmatrix}T_d\\ V_a\end{bmatrix}$$

$$y = \begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix}$$

Or equivalently

$$\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix} = \begin{bmatrix}\frac{-b}{j} & \frac{K_m}{j}\\ \frac{-K_b}{I} & \frac{-R}{L}\end{bmatrix}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix} + \begin{bmatrix}-1 & \\ & 1\end{bmatrix}\begin{bmatrix}T_d\\ V_a\end{bmatrix}$$

$$y = \begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix}$$

Thus the state space modeling matrixes are

$$A = \begin{bmatrix} [illegible] & [illegible] \\ \frac{-K_b}{L} & \frac{-R}{L} \end{bmatrix} \qquad \mathbf{B}= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 \end{bmatrix} \qquad \mathbf{D=0}$$

Now for
R=2Ω.
L=0.5H
K_m=K_b=0.1(constant)
J=0.02 kg m^2/s^2
The step response is given below (taking $\mathbf{T_d}$ =0)

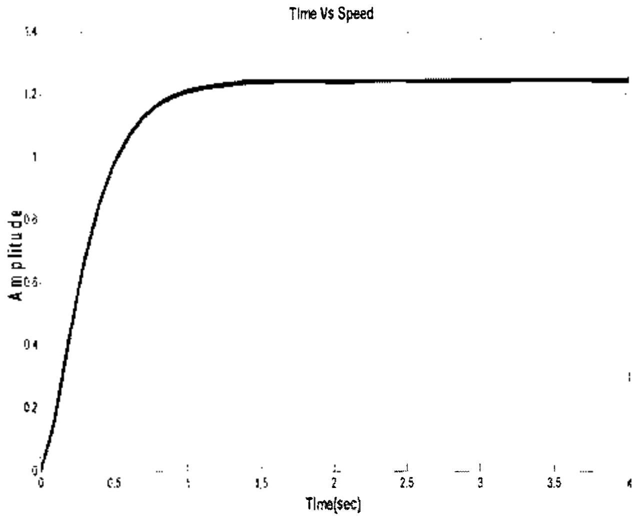


Figure 3.15: Step Response of armature controlled DC Motor using State-Space Modeling

But load torque or disturbance torque is a variable for a Dc motor and changing in a load torque in eventually affect the speed of the motor. If simulate the model of the Dc motor in time domain with distance torque the time versus speed curve will look like the below.

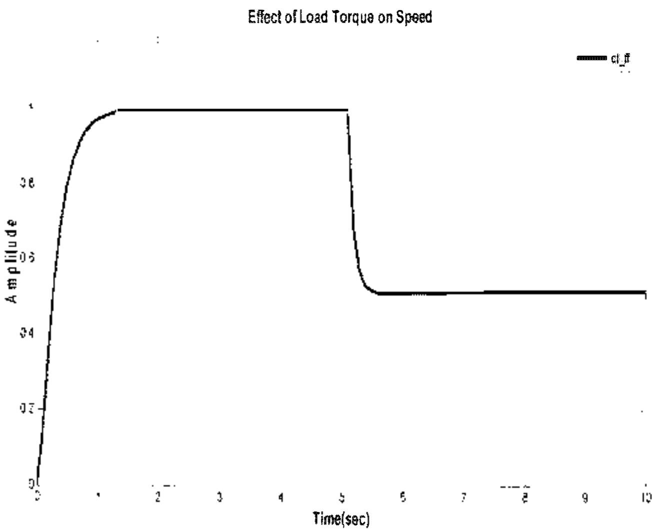


Figure 3.16: Effect of Load on Speed

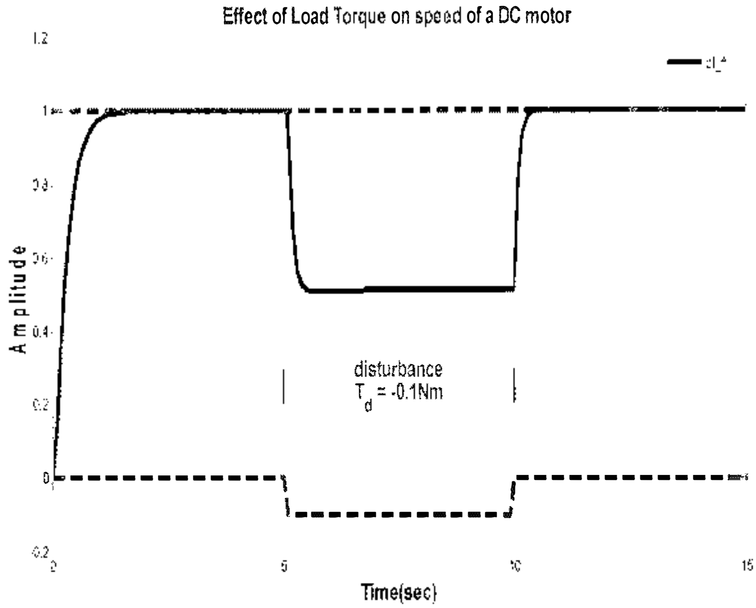
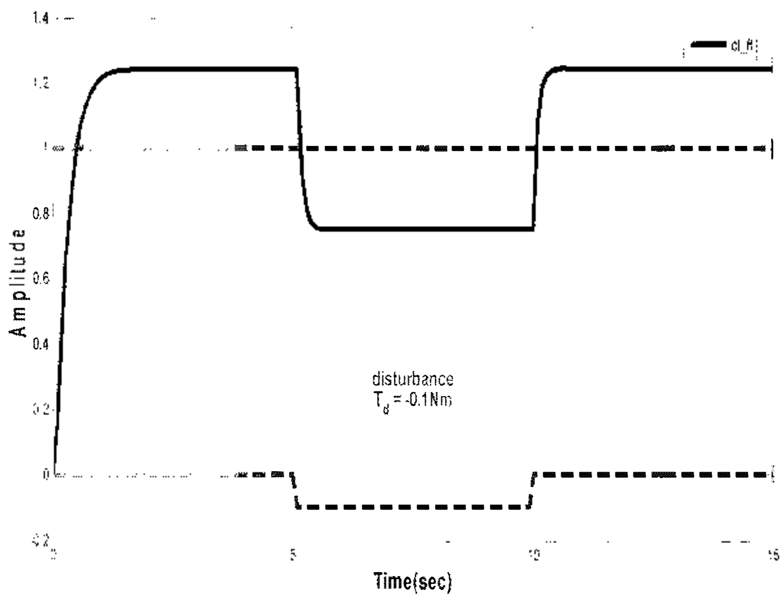


Figure 3.17: Effect of changing disturbance torque on speed

But if we use a forward gain without a feedback the speed will increase as like the theory says but the speed will not remain unchanged due to the change in disturbance torque, T_d.



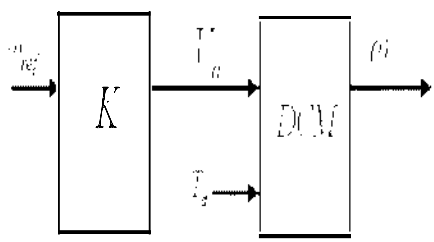


Figure 3.18: Feed-forward control of DC motor

Figure 3.19: Effect of forward gain on step response

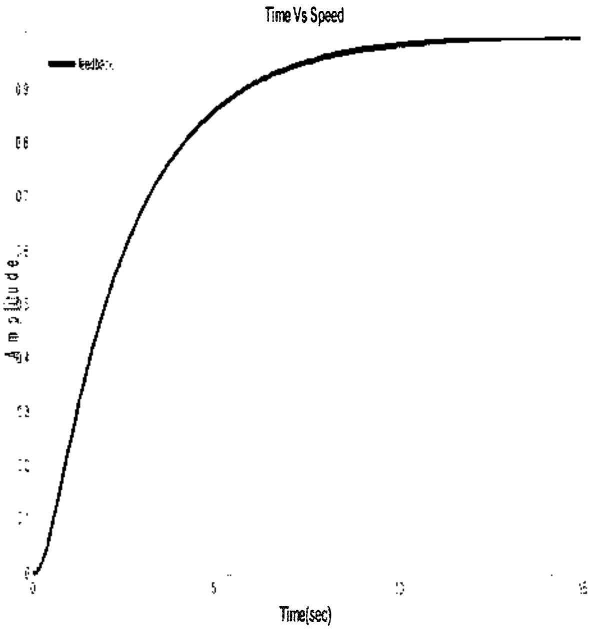


Figure 3.20: Step Response

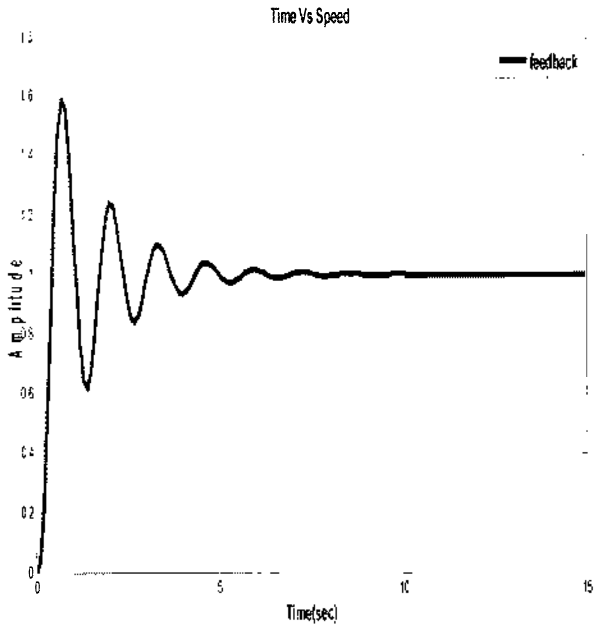


Figure3.21: Effect of big forward gain

But the purpose of maintaining the speed of the DC Motor in a constant value is not served. So this open-loop system cannot keep the speed constant while varying the disturbance or load torque.

So, to keep the speed constant we have to design a close-loop system along with a compensator which can keep the speed constant and other parameter value under control while varying the other parameter specially the disturbance torque, T_d.

To serve the purpose stated above we can use a simple unit feedback with an integral compensator or integral control along with a unit forward gain (k=1).

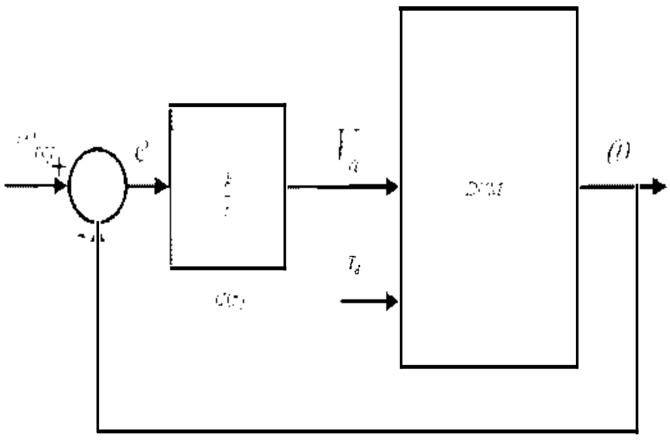


Figure 3.22: A Close loop system with an integral compensator

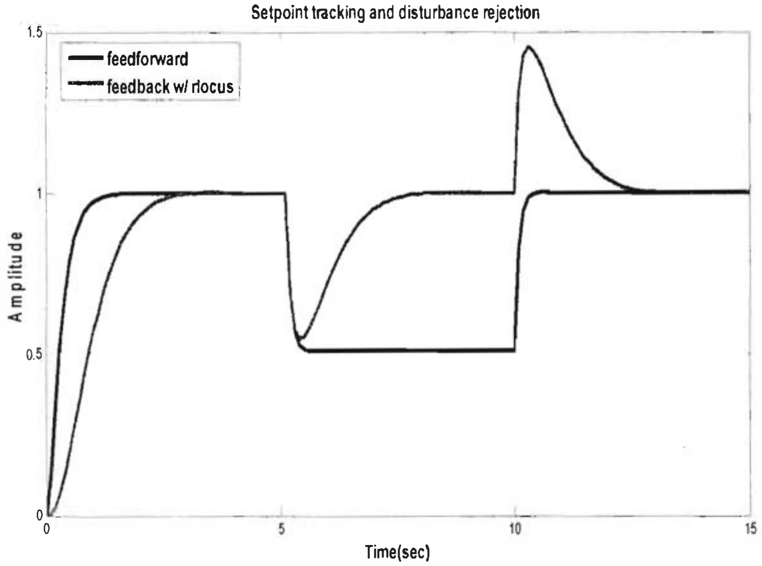


Figure 3.23: Step Response of the above system with disturbance Torque

Again the response of the system having a small value of k will result in a output alike the figure 3.24. It is clear that is not an effective on as transient time in this case is very large. So we have to increase the value of k.

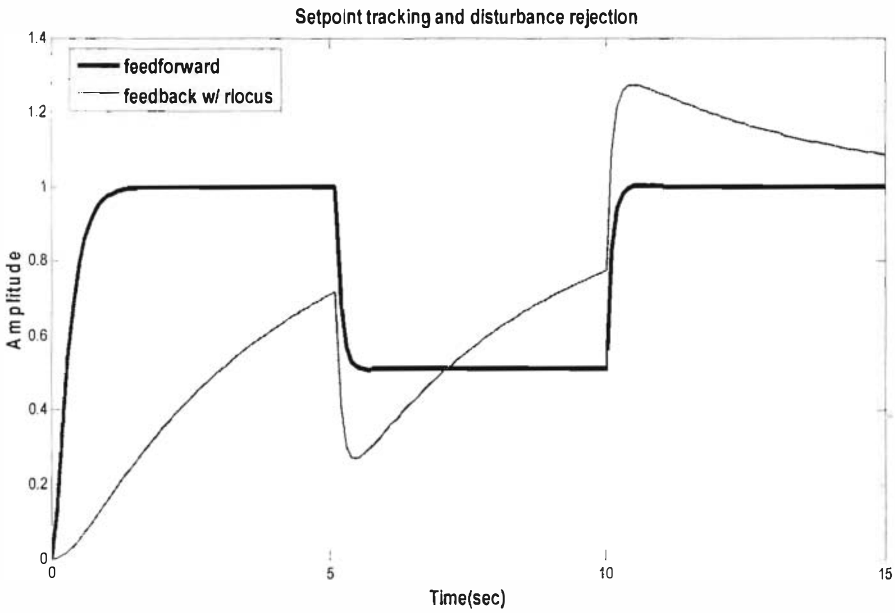


Figure 3.24: Effect of large value of K (forward gain)

As the value of K is not doing the task what it supposed to be done. This value of K results in a very large transient time for which the speed cannot be attain in its stable value in a short time. So we have to change the value or in other words increase the value of K. Randomly we chose a bigger value of K and simulate the circuit to see the variation in step response with a disturbance torque.

The response of the system for K=20 is given below.

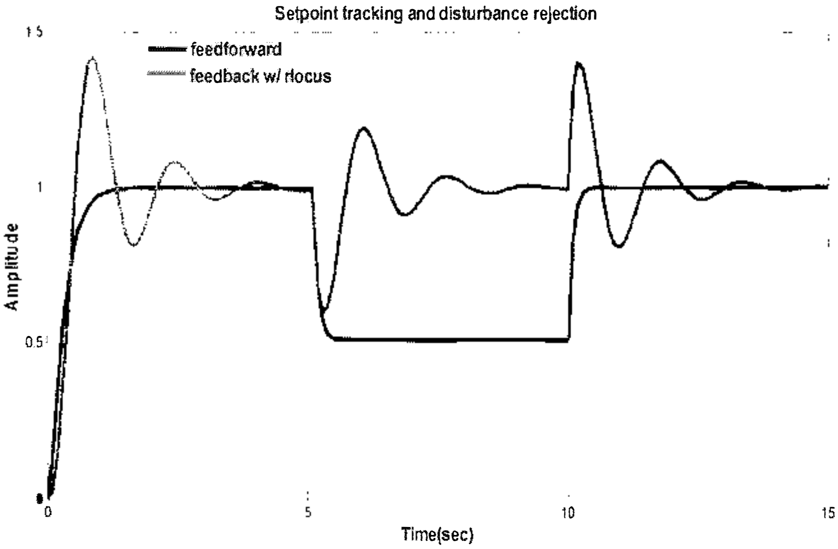


Figure 3.25: Effect of large value of K (forward gain)

This value is also not effective as change in output or speed with respect to time is not constant or in other words rippling in output occurs for a big value of K will eventually result in an unstable system. So our next task is to find out an optimum of K for which we can get a minimum stabling time and minimum ripple or without ripple.

To get an optimum value of K we can use root-locus analysis. Root-locus analysis is an analysis which is used for the controlling purpose of a DC system. It is also known as DC analysis of a system. **[8]**

3.5 Root Locus Method

The relative stability and the transient performance of a closed-loop control system are directly related to the location of the closed-loop roots of the characteristic equation in the s-plain (complex plain). It is frequently necessary to adjust one or more system parameter in order to obtain suitable root locations. Therefore, it is worthwhile to determine how the roots of the characteristics equation of a given system migrate about the s-plain as the parameter are varied.

The root-locus method was introduced by **Evan** and has been developed and utilized extensively in control engineering practice. The root-locus technique is a graphical method for sketching the locus of roots in the s-plain as a parameter is varied. In fact, the root-locus method provides the engineer with a measure of the sensitivity of the roots of the system to a variation in the parameter being considered. The root-locus technique may be used to great advantage in conjunction with the **Routh-Hurwitz** criterion. **[8]**

Analysis

In a root-locus analysis first of all the poles of a system or the roots of the characteristics equation of the system is located in the complex plain taking the forward gain, K=1. Then the value of K is changed from zero to infinity and locates the poles again and again. Thus continuing the process we get a trend of shifting the poles along an asymptote in the complex plain. The values of K for which the poles are located on the right side of the complex plain will give an unstable output and the system will be unstable. The values of K for which the poles of the system are located in the left side of the complex plain will result in a stable system. Among these values of K, a optimum value of gain K is to find out.

The root-locus analysis of the system we are talking about is given below

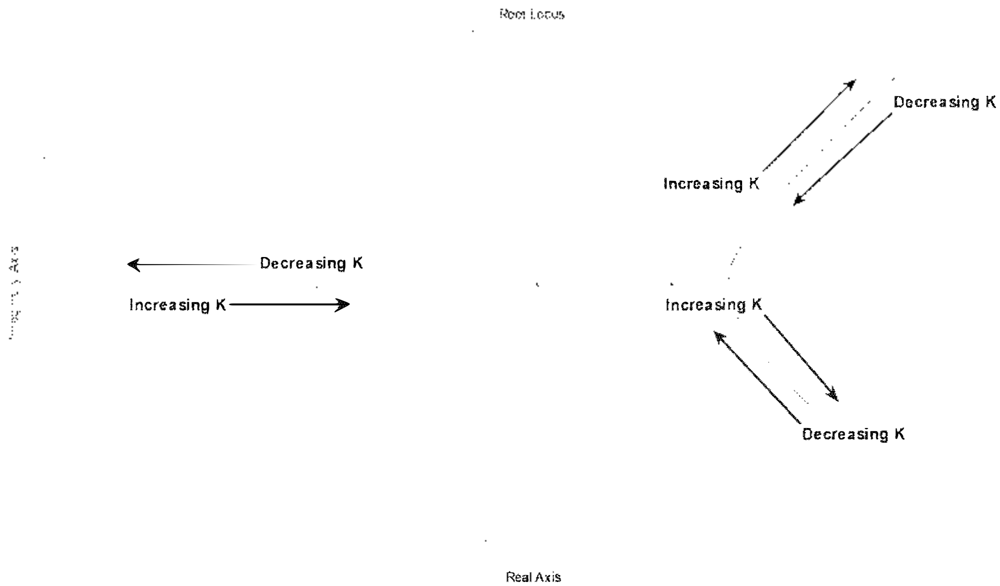


Figure 3.26: Root-Locus of the Feed- back system in figure 3.22

From the root locus (figure) we are clear that if the value of gain, K is increased two of the poles head towards the right sight of the complex plain and one pole head towards the left side of the plain along the real axis. The pole that head toward the left side with the increase of gain, k is not at all being problematic for the system or system stability. Concentration should be given to the behavior of the other two poles. For the gain value of 1, two poles stays on the left side but one pole stays at the origin of the complex plain for which the system will be unstable. So the third pole have to be pushed back to the left side for which the value of gain, K should be increased. Gain value can be increased until the poles cross the imaginary axis. But the best way to choose the optimum value of K is to choose a value for which two poles merge on one another or real axis on the left side of the complex plain.

By the theoretical calculation the optimum value of the gain, K is calculated around 4.5. For the calculation **'Routh's stability criterion'** [1] techniques has been applied to determine the range of gain, K for which system remain stable and from the range the value of the gain, K is determined by the theoretical calculation.

From the simulation result it is understood that the process that we used for keeping the speed constant is serving the purpose. But this process cannot be the best one as there are some problems arises. First of all settling time or the system from the time disturbance

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torque is introduced or applied is large. Secondly the amount of speed changed (overshoot and undershoot) during the time of disturbance torque change is also very significant. To get a smooth and robust control we have to use some other technique to do what is desired. Here **OPTIMAM CONTROL SYSTEM** can be introduced which is totally based on stat-space analysis.

3.7. OPTIMAL CONTROL SYSTEM

The design of optimal control systems is an important function in control engineering. The purpose of design is to realize a system with practical components that will provide the desired operating performance. The desired performance can be readily stated in terms of time domain performance indices. For example, the maximum overshoot and rise time for a step input are available in time domain indices. In the case of steady state and transient performance, the performance indices are normally specified in the time domain. Therefore, it is natural that we wish to develop design procedures in the time domain. **[9]**

The performance of a control system con be presented by integral performance measures. Therefore, the design of a system must be based on minimizing a performance index, such as the integral of the squared error (ISE). The systems that are adjusted to provide a minimum performance index are often called **optimal control system**. In this case, we will consider the design of an optimal control system that is described by a state variable formulation. We will consider the measurement of the state variables and their use in developing a control signal *u(t)* so that the performance of the system is optimized. **[9]**

The performance of a control system, written in terms of the state variables of a system, can be expressed in general as

$$J = \int_0^{t_f} g(x, u, t)\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

Where, **x**= the state vector, **u**= control vector and $\mathbf{t_f}$= the final time.

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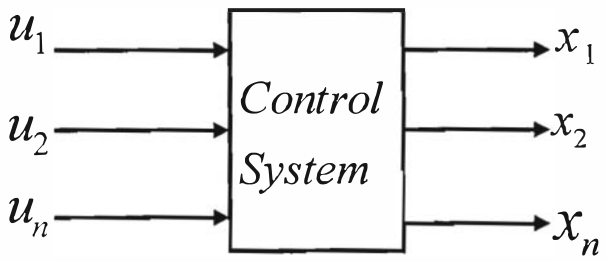

Figure 3.27: A control system in terms of input and output control variable

We are interested in minimizing the error of the system; therefore, when the desired state vector is represented as $\mathbf{x_d=0}$, we are able to consider the error as identically equal to the value of state vector. That is, we desire the system to be at equilibrium, $\mathbf{x = x_d = 0}$, and any deviation from equilibrium is considered an error. Therefore, in this case, we will consider the design of optimal control systems using state variable feedback and error **squared performance indices.**

The control system we will consider is shown in figure….. and can be represented by the vector differential equation

$\dot{x} = Ax + Bu$ 3.8

We will select a feedback controller so that **u** is some function of the measured stated variables **x** and therefore

$u = -k(x)$ 3.9

For example, we might use

$u_1 = -k_1x_1 \quad u_2 = -k_2x_2$............................and $u_m = -k_mx_m$

Alternatively, we might choose the control vector as

$u_1 = -k_1(x_1 - x_2) \qquad u_2 = -k_2(x_2 + x_3)$

The choice of the control signals is somewhat arbitrary and depends partially on the actual desired performance and the complexity of the feedback structure allowable. Often, we are limited in the number of state variables available for feedback, since we are only able to utilize measurable state variables.

In our case, we limit the feedback function to a linear function so that $u = -kx$, where k is an m×n matrix, therefore, in expanded form we have

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = -\begin{bmatrix} k_{11} & \cdots & k_{1n} \\ \vdots & & \vdots \\ k_{m1} & \cdots & k_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Substituting equation 3.8 into equation 3.9, we obtain

$$\dot{x} = Ax - BKx = Hx \quad \ldots\ldots \quad 3.10$$

Where **H** is the n×n matrix resulting from the addition of the element of **A** and **–BK.**
Now, returning to the error squared performance, the index for a single state variable, x_1, is written as

$$J = \int_0^{t_f} [x_1(t)]^2 \, dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3.11$$

A performance index written in terms of two state variables would then be

$$J = \int_0^{t_f} [x_1(t)^2 + x_2(t)^2] \, dt \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.12$$

Since we wish to define the performance index in terms of an integral of the sum of the state variable squared, we will utilize the matrix operation

$$x^T x = [x_1 \; x_2 \; \ldots \; x_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = (x_1^2 + x_2^2 + \ldots + x_n^2)$$

where $\mathbf{x}^T$ indicates the transpose of the **x** matrix. Then the specific form of the performance index, in terms of state vector, is

$$J = \int_0^{t_f} (x^T x) \, dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{3.13}$$

Considering the above equation, we will let the final time of interest be $t_f = \infty$. To obtain the minimum value of J, we postulate the existence of an differential so that

$$\frac{d}{dt}(x^T P x) = -x^T x$$

Where **P** is to be determined. A symmetric **P** will be used to simplify the algebra without any loss of generality. Then, for a symmetric **P** matrix, $p_{ij} = p_{ji}$. Completing the differentiation indicated on the left side of the above equation we have

$$\frac{d}{dt}x^T P x = \dot{x}^T P x + x^T P \dot{x} \quad \ldots\ldots 3.14$$

Substituting equation 3.10, we obtain

$$\frac{d}{dt}x^T P x = (Hx)^T P x + x^T P (Hx)$$

$$= H^T x^T P x + x^T P H x$$

$$= x^T (H^T P - PH) x$$

if we let $H^T P - PH = -I$ then above equation becomes

$$\frac{d}{dt}x^T P x = -x^T x \quad \ldots\ldots 3.15$$

which is the exact differential we are seeking. Substituting the above Equation into Equation 3.13 we obtain

$$J = -\int_0^\infty \frac{d}{dt}x^T P x \, dt = -x^T P x \Big|_0^\infty = -x^T(\infty) P x(\infty) + x^T(0) P x(0) \quad \ldots\ldots 3.16$$

In the evaluation of the limit at t = ∞, we have assumed that the system is stable, and hence x(∞) = 0, as desired. Therefore to minimize the performance index *J*, we consider two equations

$$J = -\int_0^\infty \frac{d}{dt}x^T P x \, dt = x^T(0) P x(0) \ldots \quad (3.17)$$

$(H^T P + PH = -I)$. (3.18)

So, the steps are then as follows

1. Determining the matrix P that satisfies Equation (3.17), where H is known.
2. Minimize J by determining the minimum of the Equation (3.18) by adjusting one of more unspecified system parameters.

For the performance index

$$J = \int_0^{\infty} (x^T I x + \lambda u^T u)\, dt \ldots\ldots \quad 3.19$$

Two equations are

$$J = x^T(0)\, P x(0)$$

$$H^T P + PH = -Q$$

Where, $Q = (I + \lambda K^T K)$.

For the performance index

$$J = \int_0^{\infty} (x^T Q x + R u^2)\, dt \ldots\ldots\ldots \quad 3.20$$

Where, R is a scalar weighting factor. This index is minimized when

$$K = R^{-1} B^T P \quad \ldots\ldots\ldots \quad 3.21$$

Then n×n matrix **P** is determined from the solution of the equation

$$A^T P - PA - PBR^{-1}B^T P - Q = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{3.22}$$

3.7. LQR (Linear Quadratic Regulator)

Linear quadratic Regulator is an optimal control of a system for which the performance is minimized using a performance index integral equation. This control technique is applicable to a linear system and the integration of squared error is minimized by using this technique taking different parameter in consideration. So this technique is named as **Linear Quadratic Regulator (LQR). [10]**

Performance index stated below is to be minimized by this technique,

$$J = \int_0^{\infty} (x^T Q x + R u^2)\, dt$$

Where, R is a scalar weighting factor. This index is minimized when

$$K = R^{-1} B^T P$$

Then n×n matrix **P** is determined from the solution of the equation

$$A^T P - PA - PBR^{-1}B^T P - Q = 0$$

This performance index is often known as **the quadratic cost function.**

The quadratic cost function or performance with output weighting is written as

$$J = \int_0^{\infty} (x^T Q x - u^T R u - 2y^T N y)\, dt \; \ldots\ldots \qquad \ldots\ldots \qquad 3.23$$

Double feedback control using LQR technique

Double feedback control for the scheme stated in figure….. using LQR technique is the next task to get the desired output. At first we have to model the DC motor using state space modeling technique. The state space modeling of a armature controlled DC motor can be written as, **[10]**

$$\frac{d}{dt}\begin{bmatrix}\omega(t) \\ i(t)\end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K_t}{J} \\ -\frac{K_b}{L} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix}\omega(t) \\ i(t)\end{bmatrix} - \begin{bmatrix} -1 & \\ & 1 \end{bmatrix}\begin{bmatrix} T_d \\ V \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix}\omega(t) \\ i(t)\end{bmatrix}$$

Or equivalently

$$\frac{d}{dt}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix} = \begin{bmatrix} \frac{k}{J} & \frac{K}{J} \\ \frac{-K}{l} & \frac{-R}{L}\end{bmatrix}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix}T_d\\ V_a\end{bmatrix}$$

$$y = \begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix}$$

Thus the state space modeling matrixes are

$A = [\ldots]$ B= $\begin{bmatrix}1\\1\end{bmatrix}$ $C = \begin{bmatrix}0 & 0\end{bmatrix}$ **D=0**

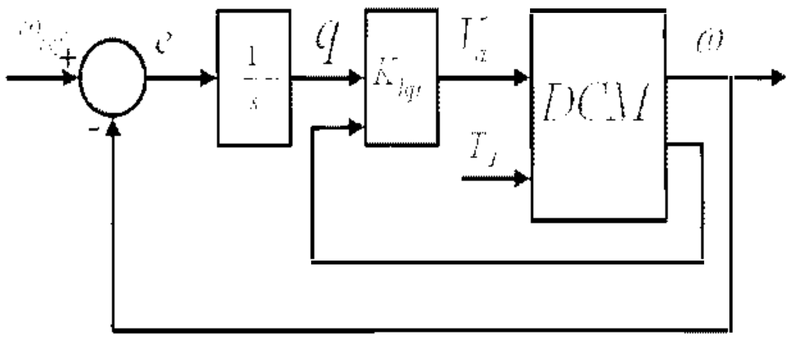

Figure 3.28: LQR control Scheme of a DC motor

Next step is to find out the state space modeling of the DC Motor with a integrator in series with it. It can be done by a MATLAB code, [1 ; tf(1,[1 0])] * dcm(1) , where dcm(1) is the state space modeling of the DC Motor.

Then using the matrix A, B, C and D and the Equation stated below we can determine optimum values of K.

$H^T P + PH = -Q$

$K = R^{-1} B^T P$

$A^T P - PA - PBR^{-1} B^T P - Q = 0$

From the above three equation, first of all the vector **P** is to be find out then **Q** and finally **K**. Here R is a scalar and is given as specified.

Rejection of Load torque disturbance by a close-loop system for the vector K_{lqr} having small value

If the value of forward gain of the figure 3.29 is very large then the output has some rippling as illustrated in figure 3.30. it can also be explained by the root-locus analysis method. Large values of forward gain K_{lqr} gives an output that stables with a very short amount of time but has some overshoot and undershoot while changing load (or disturbance torque). But the system gives a fixed speed (or output). This overshoot or undershoots can be problematic in practical cases. So we cannot take a large value of K_{lqr} while controlling the speed to be constant.

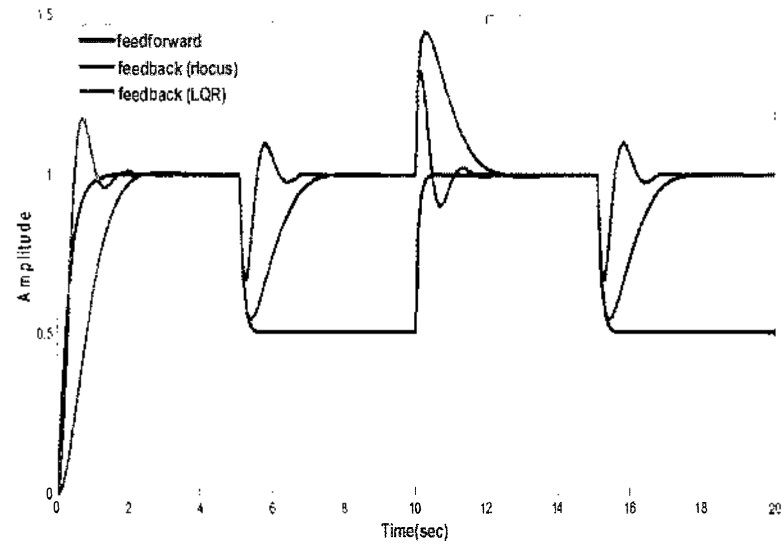

Figure 3.30: Rejection of Load torque disturbance by a close-loop system for the vector K_{lqr} having large value

If we take a small value for the controlling purpose then the output will look like as illustrated below in the Figure 3.31

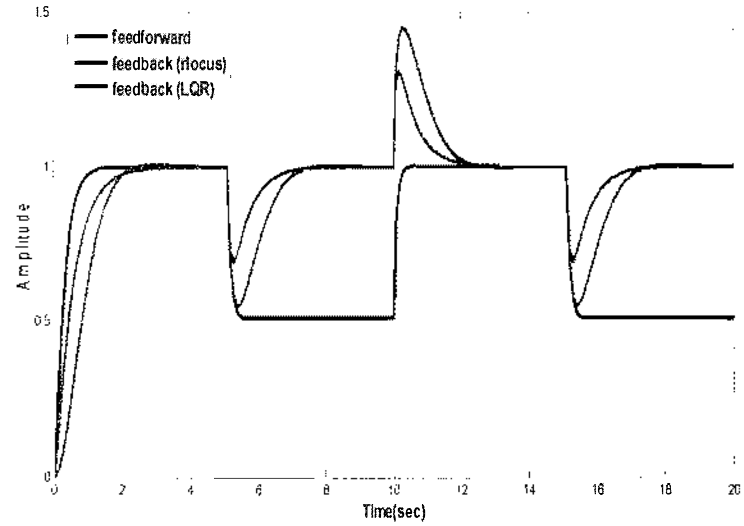

Figure 3.31: Rejection of Load torque disturbance by a close-loop system for the vector K_{lqr} having small value

For small values of K_{lqr} overshoot and undershoot is lessen but the settling time to get the desired speed is increased which can be problematic in some practice case where there is frequent change in load or disturbance torque.

Rejection of Load torque disturbance by a close-loop System (LQR) for the optimum value of K_{lqr}

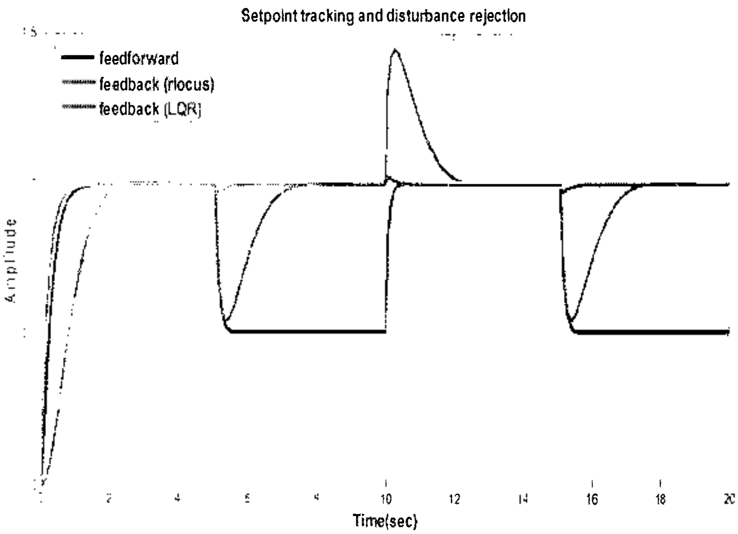

Figure 3.32: Rejection of Load torque disturbance by a close-loop system for the vector K_{lqr} having optimul value

By using the LQR (linear Quadratic Regulator) method we calculate optimum values of K_{lqr} which gives the best result to keep the speed constant. This method is very useful for our controlling purpose.

LQR method can give us a controlling scheme which is very useful to get a constant speed while changing load torque frequently. The result is illustrated in the Figure 3.32.

Conclusion:

It is noted that the project is on the speed control of DC motor. The characteristics of the various types of DC motor are discussed in this project. But it is given more focus on separately excited DC motor to control the speed. Two parts are included to control the speed of a DC motor such as control systems and electrical analysis. Different parameters (resistance, voltage, current etc) are used to control the speed of a DC motor. It is totally based on theoretical analysis. Math lab software, Root-Locus analysis and LQR system are efficient parameters to perform this project accurately.

References

[1]. B.L.THERAJA & A.K.THERAJA.A TEXT BOOK OF ELECTRICAL TECHNOLOGY IN.S.I UNITS, Volume II, AC & DC MACHIN (2006)

[2]. Muhammad H. Rashid, Third Edition (2003), Power Electronics,.

[3]. Richard C. Dorf and Robert H Bishop Modern Control system,. Tenth Edition, 2008

Chapter 2, Mathematical Models of System.

Chapter 11,The Design of State Variable Feedback Systems

Chapter 15, DC Drives.

[4]. Katsuhiko Ogata. Modern Control Engineering, Fourth Edition,2004

Chapter 1, Introduction to Control Systems

Chapter 3, Mathematical Modeling of Dynamic System

[5]. http://www.howstuffworks.com/motor.htm

[6]. Norm and Nice, Control System Engineering, Fifth edition, 2004.

[7]. dsp.vscht.cz/konference_matlab/...thondel/vondrich_thondel.pdf

[8]. claymore.engineer.gvsu.edu/~jackh/books/model/.../linear.pdf

[9]. J. Vondřich, E. Thőndel. MODELLING OF LQR CONTROL WITH MATLAB

[10]. Jo~ao P. Hespanha, LQG/LQR controller design, ,April 1, 2007

APPENDIX A

1. MATLAB Code for the step Response of armature controlled DC Motor

```
clear all
clc
km=0.1;
jm=0.02;
Ra=2;
b=0.01;
kb=0.1;

num=[km];
den=[Ra*jm Ra*b+kb*km];
y=tf(num,den);
z=step(y);
plot(z)
title('Time Vs Speed')
xlabel('Time')
ylabel('Speed')
```

2. MATLAB Code for the step Response of Field controlled DC Motor

```
clear all
clc
Km=50e-3;
Jm=10e-3;
La=2;
Ra=0.5;
b=0.01;
```

```
kb=0.01;
km=200;
num=[km];
den=[La*Jm Ra*Jm+La*b Ra*b+kb*km];
y=tf(num,den);
t=linspace(0,10,1000);
z=step(y,t);
plot(t,z)
title('Speed varrying due to resistance & inductance value')
xlabel('Time')
ylabel('Speed')
```

3. MATLAB Code of effect of Disturbance on step response

```
clear all
clc
R = 2.0;            % Ohms
L = 0.5;            % Henrys
Km = 0.1; Kb = 0.1;     % torque and back emf constants
Kf= 0.2;            % Nms
J = 0.02;           % kg.m^2/s^2
h1 = tf(Km,[L R]);          % armature
h2 = tf(1,[J Kf]);          % eqn of motion

dcm = ss(h2) * [h1 , 1];      % w = h2 * (hl *Va + Td)
dcm = feedback(dcm,Kb,1,1);   % close back emf loop

Kff = 1/dcgain(dcm(1));
```

```
t = 0:0.1:15;
Td = -0.1 * (t>5 & t<10);      % load disturbance
u = [ones(size(t)) ; Td];      % w_ref=1 and Td

cl_ff = dcm * diag([Kff,1]);    % add feedforward gain
set(cl_ff,'InputName',{'w_ref','Td'},'OutputName','w');

h = lsim(cl_ff,u,t);
plot(t,h,t,u,'--')
xlabel('Time(sec)')
ylabel('Amplitude')
title('Setpoint tracking and disturbance rejection')
legend('cl\_ff')

% Annotate plot
line([5,5],[.2,.3]); line([10,10],[.2,.3]);
text(7.5,.25,{'disturbance','T_d = -0.1Nm'},...
        'vertic','middle','horiz','center','color','r');
```

4. **MATLAB Code of Effect of field Resistance and Load on speed**

```
clear all
clc
km=50e-3;
Rf=linspace(1e-3,5e-3,10);
b=0.5;
tau_f=1e-3;
tau_l=100e-3;
```

```
jm=linspace(0.5e-3,1e-3,10);
Lf=5;
R=5;

for i=1:length(jm)

num11=km/(jm(i)*Lf);
num12=1;
den11=[1 b/jm(i)];
den12=[1 R/Lf];
sys11=tf(num11,den12);
sys12=tf(num11,den12);
sys=sys11*sys12;
out1=step(sys);
figure(1)
plot(out1)
title('Speed VS Load')
hold on
grid on
end

for i=1:length(Rf)
num21=km/(b*Rf(i));
num22=1;
den21=[tau_f 1];
den22=[tau_l 1];
sys21=tf(num21,den21);
```

```
sys22=tf(num22,den22);
sys=sys21*sys22;
out2=step(sys);
figure(2)
plot(out2)
title('Speed VS Field current')
hold on
grid on
end
```

5. **MATLAB Code of Effect of Voltage on speed**

```
clear all
clc
syms t;
x=0:10;
for i=1:length(x)
km 50e-3;
Rf=3;
b=0.5;
tau_f=1e-3;
tau_l=100e-3;
jm=1e-3;
Lf=5;
num11=km/(jm*Lf);
num12=1;

den11=[1 b/jm];
den12=[1 Rf/Lf];
```

```
sys11=tf(num11,den12);
sys12=tf(num11,den12);
sys1=sys11*sys12;
[u,t] = gensig('square',70,70,0.01);
y=lsim(sys1,u*x(i),t);
plot(t,y)
hold on
end
title('effect on speed while varying voltage')
```

6. MATLAB Code of Effect of armature Resistance on speed

```
clear all
clc
km=50e-3;
kb=5e-6;
Ra=[1e-3 2e-3 3e-3 4e-3 5e-3];
b=0.5;
jm=1e-3;

for i=1:length(Ra)
   tau=Ra(i)*jm/(Ra(i)*b+kb*km)
   num1=km/(Ra(i)*b+kb*km);
   den1=[tau 1];
   sys=tf(num1,den1);
   out1=step(sys);
   figure(1)
   plot(out1)
```

```
   hold on
   grid on
end
 title('Speed changes due to armature resistance')
xlabel('Time')
ylabel('Speed')
```

7. **MATLAB Code of Step Response taking La/Ra in consideration.**

```
Km=50e-3;
Jm=10e-3;
La=2;
Ra=linspeace(0.5,2,10);
b=0.01;
kb=0.01;
km=200;
num=[km];
den=[La*Jm Ra*Jm+La*b Ra*b+kb*km];
y=tf(num,den);
t=linspace(0,10,1000);
z=step(y,t);
plot(t,z)
```

8. **MATLAB Code for Setpoint tracking and Load Distrubance Rejection By Root-Locus Method**

```
clear all
clc
R = 2.0;            % Ohms
L = 0.5;            % Henrys
Km = 0.1; Kb = 0.1;     % torque and back emf constants
Kf = 0.2;            % Nms
```

```
J = 0.02;              % kg.m^2/s^2
h1 = tf(Km,[L R]);           % armature
h2 = tf(1,[J Kf]);           % eqn of motion

dcm = ss(h2) * [h1 , 1];     % w = h2 * (h1*Va + Td)
dcm = feedback(dcm,Kb,1,1);  % close back emf loop

Kff = 1/dcgain(dcm(1));
t = 0:0.1:15;
Td = -0.1 * (t>5 & t<10);      % load disturbance
u = [ones(size(t)) ; Td];      % w_ref=1 and Td

cl_ff = dcm * diag([Kff,1]);   % add feedforward gain
set(cl_ff,'InputName',{'w_ref','Td'},'OutputName','w');

K = 5;
C = tf(K,[1 0]);           % compensator K/s

cl_rloc = feedback(dcm * append(C,1),1,1,1);
H1= lsim(cl_ff,u,t);
H2= lsim(cl_rloc,u,t);
plot(t,H1,t,H2)
set(cl_rloc,'InputName',{'w_ref','Td'},'OutputName','w');
xlabel('Time(sec)')
ylabel('Amplitude')
title('Setpoint tracking and disturbance rejection')
legend('feedforward','feedback w/ rlocus','Location','NorthWest')
```

9. MATLAB Code for Setpoint tracking and Load Distrubance Rejection By LQR Method

```
clear all

clc

R = 2.0;              % Ohms

L = 0.5;              % Henrys

Km = 0.1; Kb = 0.1;   % torque and back emf constants

Kf = 0.2;             % Nms

J = 0.02;             % kg.m^2/s^2

h1 = tf(Km,[L R]);          % armature

h2 = tf(1,[J Kf]);          % eqn of motion


dcm = ss(h2) * [h1 , 1];     % w = h2 * (h1*Va + Td)

dcm = feedback(dcm,Kb,1,1);  % close back emf loop


Kff = 1/dcgain(dcm(1));

t = 0:0.1:15;

Td = -0.1 * (t>5 & t<10);     % load disturbance

u = [ones(size(t)) ; Td];     % w_ref=1 and Td


cl_ff = dcm * diag([Kff,1]);   % add feedforward gain


K = 5;

C = tf(K,[1 0]);          % compensator K/s


cl_rloc = feedback(dcm * append(C,1),1,1,1);
```

```
dc_aug = [1 ; tf(1,[1 0])] * dcm(1); % add output w/s to DC motor model

K_lqr = lqry(dc_aug,[1 0;0 20],0.01);

P = augstate(dcm);                    % inputs:Va,Td  outputs:w,x
C = K_lqr * append(tf(1,[1 0]),1,1);  % compensator including 1/s
OL = P * append(C,1);                 % open loop

CL = feedback(OL,eye(3),1:3,1:3);     % close feedback loops
cl_lqr = CL(1,[1 4]);

H1 = lsim(cl_ff,u,t);
H2 = lsim(cl_rloc,u,t);
H3 = lsim(cl_lqr,u,t);

plot(t,H1,t,H2,t,H3)
xlabel('Time(sec)')
ylabel('Amplitude')
title('Setpoint tracking and disturbance rejection')
legend('feedforward','feedback (rlocus)','feedback (LQR)','Location','NorthWest')
```