Univariate Time Series Forecasting: A Study on Monthly Tax Revenue of Bangladesh

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Abstract

In recent years, Bangladesh has taken various steps to modernize tax system in order to enhance tax effort. Due to subsequent increase in financial constraints globally, economy's reliance on domestic resource mobilization continues to intensify. As a result, tax revenue target for every forthcoming budget appears to be buoyant albeit the prevalence of domestic constraints, i.e. inefficiencies in tax system, narrower tax base along with numerous exemptions and political instability. To enhance tax effort to reduce fiscal vulnerability, a neoteric revenue forecasting procedure is necessary. But, in Bangladesh, during the budget preparation, the method to target tax revenue is based on the growth rate extended with discretionary adjustments for a number of updated assumptions and personal judgments, which can lead to huge forecast error. This exercise attempts to identify an appropriate model by scrutinizing three approaches - ARIMA SARIMA multiplicative approach, Holt-Winters seasonal multiplicative approach and Holt-Winters seasonal additive approach - to forecast monthly tax revenue of Bangladesh and finds that, Holt-Winter seasonal multiplicative approach is the most appropriate method with minimum forecast error.

Keywords: Tax revenue forecasting, Box-Jenkins Method, ARIMA, SARIMA. Holt-Winters seasonal multiplicative approach, Holt-Winters seasonal additive approach.

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I. Introduction

Over the years, one of the key issues in the design of a sound fiscal policy has been the accuracy of budget forecast, particularly tax revenue forecast. For a sound fiscal environment with lower arrears to pursue a consistent Medium-term budget framework for any economy, accurate and relevant revenue forecasting method is crucial to avoid unexpected revenue shortfall or to set target for revenue collection before budget preparations. Large forecast errors from poorly performing forecasting procedures can lead to significant budget management problems, expenditure arrears and stop-and-go expenditure policies, which can pose severe obstacle to the development of a realistic medium-term budget plan.

In Bangladesh, Ministry of Finance (MOF) is the supreme authority to target tax revenue. Before every budget preparation, Ministry of Finance (MOF) sets tax revenue as a budgetary target, but tax effort depends on the accuracy in forecasting method and the institutional efficiency of tax collection authority, i.e. National Board of Revenue, Bangladesh (NBR). Thus, favorable tax revenue performance is determined by the consistent coherence performance of these two institutions simultaneously. Over the last five years, Bangladesh has performed well to enhance its tax revenue efforts; though it is still low compared to other similar countries from the perspective of economic development; Bangladesh is even in the bottom half among South Asian countries on this context (Appendix A. Table 01).

Though Bangladesh is on the right track in meeting tax revenue targets; the gap between targeted and actual tax revenue still exists which are fairly large and remains to be volatile in its pattern. Based on the tax revenue data reported by Ministry of Finance, Bangladesh, it has been calculated that the average tax gap[4] lies between 8-12 billion taka for the fiscal year 2005 to 2012 (Appendix B Figure 01). These patterns have been attributed to weaken institutional capacity to set targets, which may be one of the causes of inaccuracy of tax revenue projection during pre-budget.

Tax revenue buoyancy in Bangladesh is still more than unity, tax efforts is below unity, which explains Bangladesh has the potential to enhance tax effort through reforming the tax system.

[4] Tax gap is defined as the difference between targeted and actual tax revenue.

Although government of Bangladesh has taken various initiatives to modernize tax system through tax automation; lack of institutional capacity to collect tax revenue is still one of the greatest challenges to meet.

This may be one of the logical reasons behind volatility in tax revenue collection, which brings huge gaps between projected tax revenue figures in budget and actual tax revenue collection. And the huge gaps between actual and targeted figures reported in budgetary documents can become a severe obstacle to achieve sustainable fiscal management and to improve domestic resource mobilization.

It is surprising that, tax revenue forecasting techniques in Bangladesh are generally not put down in formal documents, and country practices are often a mix of idiosyncratic budget practices and influences from legacy systems. Not a single remarkable exercise has been carried out in identifying appropriate methodology of revenue forecasting from those institutions involved in revenue collections in Bangladesh.

Methods for tax revenue forecasting are not entirely free from errors. Some economic factors, like growth variations in different economic sectors, international vulnerability, inflation, influence of political factors on tax policy changes (exemptions, unequal treatments) can ascribe to the unpleasant patterns of tax revenue (Abed, 1998)[5]. In addition, underdeveloped institutional capacities can be some of the likely reasons behind intentionally overstated forecasts in developing countries (Lienert and Sarraf, 2001)[6].

This study attempts to find an appropriate forecasting model through analyzing various time series forecasting models and quantifying the gaps between actual and forecasted values for these models. Finally, this paper compares the tax gaps between actual and forecasted values calculated by Ministry of Finance (MoF) to tax gaps calculated from appropriate forecasting model.

II. Literature Review

Like all other forecasting literatures in economic theory, tax revenue forecasting is done following some common assumptions.

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[5] During 1985 –1995, tax revenue forecasts were above actual values for about 77 percent of the time in a sample of 34 low-income countries. He argued that tax policy changes in different regime of the government leads to major discrepancies in forecasting model.

[6] Forecasting fluctuations are explained by government corruption and which is motivated by the well-established empirical fact of a high state captures in low-income countries.

These assumptions are consistent with variables like growth in the national income, inflation rate and interest rate. Although there is a dearth of forecasting exercises in Bangladesh, a few worth mentioning studies related to tax revenue forecasting are highlighted in this study.

In a study, Kairala (2011) used seasonal ARIMA and exponential seasonal smoothing method and winter models to scrutinize the forecast revenue of Nepal which pursues an erratic movement along time- there were over-estimation of revenue followed by under-estimation. He pointed out that SARIMA model was superior to any other forecasting models. He also argued that existing models of revenue forecasting in Nepal were constructed on the basis of growth rate; resulting in frequent higher discrepancies in the estimation.

In an attempt Fullerton (1991) applied univariate ARIMA model integrated with a composite method of sales tax revenue to forecast using quarterly revenue data. This study suggested that a composite model based on univariate ARIMA projections of Idaho retail sales tax receipts provided better forecasts than either single model. He also posited that given any existing efficient institutional capacity, forecast error appears because of some undesired occurrences in both external and internal factors intrinsic to tax system.

Danninger (2005) applied the Principal-Agent framework[7] based on the tax structure of Azerbaijan to diagnose the systematic relationship between tax effort efficiencies and incentives to tax collection agencies. He attempted to argue that, upward bias in forecasts is the result of a government's inability to monitor the performance of its tax administration.

Schoefish (2005) introduced a more general forecasting model[8] based on the assumption that tax elasticity must vary depending on the phase of economic cycle.

[7] The standard solution to the principal-agent problem is the design of an incentive compatible contract for the agent (e.g., Holmström and Bengt, 1979; Grossmann and Hart, 1983), which links compensation to an observable variable varying with the principle's objective function and thus counterbalances the agent's conflicting goals. In the given scenario, this would suggest that the compensation of the revenue administration should be linked to the revenue collection performance (e.g., a fixed share of collected revenue is distributed as a bonus). In reality, however, such contracts are not practical, as they would be costly and face serious political opposition. First, compensation schemes would likely be expensive to discourage individual rent taking as targeting would be a problem. Second, they would be inefficient, since the role of other factors affecting revenue, such as economic growth, is quite large. Finally, they would be hard to justify politically, as their prime function is to reward non-corrupt behavior.

[8] Yt = α+xtB0+xt-1B1+....+xt-pBp +et, which permits that tax base (xt)to be a moving combination of current and past values of xt . He also used the lagged values of Yt explanatory variables. If absolute value of $\beta = 1$, then tax revenue growth rates ΔlogYt follow a trend plus errors in this model. Alternatively, if the estimated absolute value of $\beta < 1$, then, the impacts of the previous period's tax revenue on current revenue follows relatively smaller effect and it would be declining patterns for the distant past time period.

He also included seasonal factors in tax revenue collection for monthly and quarterly data. He used tax base as proxy to the moving geometric combinations of one or more macroeconomic regressors and their lagged values. He also considered the growth rates in place of tax base and concluded that they were all necessary to calculate good forecast values.

Legeida and Sologoub (2003) applied a stationary time series approach and established a stable long-term relationship between VAT (Value Added Tax) base[9] and VAT productivity[10]. Finally, they attempted to apply the ARIMA model for monthly data to forecast VAT revenue in short-run. They concluded that ARIMA is fully consistent with the government projections for the budget. They also argued that, VAT refund, debt, numerous tax exemptions and extremely low VAT compliance might complicate the forecast of VAT revenue in Ukraine.

Chowdhury and Hossain (1988) examined the tax structure of Bangladesh to estimate tax elasticity. This study showed that overall tax structure is inelastic with respect to national income. This study also identified that tax yield can be increased by removing various exemptions, smoothing existing multiple tax rates and improving tax administration capacity. They also mentioned that, to project higher tax yield in upcoming budget it is required to expand tax base and improve tax administration capacity.

Ahmed (2012) conducted a study using ARCH model and coefficient of variations to explain the volatility in the flow of tax revenue against periodical changes in different tax series in Bangladesh. The result of this study pointed that most of the tax series have significant high level of volatility in both short and long run during this cluster of periods.

III. Bangladesh: Revenue Structure and Performance

After independence revenue share of all categories of tax revenue has increased gradually in Bangladesh, initially custom duty had larger share but its share continued to fall till date.

[9] VAT base has been calculated by including all industrial and service sector, agricultural sector are excluded in Ukraine.

[10] VAT productivity represents the efficiency of any tax system. It is measured simply by a ratio of the VAT revenue to GDP ratio to the Standard VAT rate. Sometimes, VAT revenue to total consumptions is taken into consideration to calculate VAT productivity in place of VAT to GDP ratio. Higher ratio indicates that given standard rate efforts is high resulting more institutional efficiency underlying constant tax and economic factors in any country.

Like other developing countries, tax structure of Bangladesh is highly dominated by indirect taxes; mainly value added taxes (VAT) and custom duty. Domestic VAT has a dominant role in this increase (Appendix B Figure 02). Higher buoyancy in domestic demand causes more domestic economic activities, some remarkable institutional reforms[11] in tax structure through various amendments in tax policies by National Board of Revenue (NBR) contributed to enhance domestic VAT revenue effort. In the last five years, on an average, domestic VAT revenue registered a ninety percent rate of growth.

Although indirect tax revenue has greater share, direct tax revenue is also gradually increasing due to the government's continuing tax modernization initiatives, but the share of direct tax revenue of total tax revenue is still lower compared to indirect tax revenue as the latter is increasing at a greater rate than the earlier one (Ahsan et al, 2011) . Share of indirect tax revenue accounts for 70 percent, while direct tax revenue possesses 30 percent of total NBR taxes. In spite of higher growth in income tax during the last five years, shares of direct tax revenue seems to be quite unimpressive because of the lower tax nets with higher distortions, income tax incentives, lack of significant numbers of TIN holders, and also lower corporate tax base.

While indirect tax revenue accounts for the lion's share of total tax revenue, it is evident from data that domestic based tax revenue increases at a faster rate compared to import based tax revenue. So, tax performance in Bangladesh has become more dependent on domestic based tax revenue. But, this may not be an ideal tax structure as it is apparent that share of direct tax is still far lower than the share of indirect tax revenue. Sometimes, higher share of indirect taxes may be a cause of less equitable distribution, higher inflation. In spite of the upward trend in tax revenue growth rate, tax to GDP ratios does not increase more given the rates of inflation and economic growth. As a result, tax to GDP ratio is still significantly lower among similar countries (in terms of economic structure and performances) in South Asian, even than Nepal and Bhutan. Average tax effort index[12] of Bangladesh is 0.51; this indicates that Bangladesh has a lot of potentiality to enhance tax efforts by taking reforming measures in the existing tax system.

[11] Widening of VAT net to the wholesale and retail stages and the inclusion of some services; Strengthening VAT administration, the management of VAT audit and investigation has been strengthened with technical assistance from the DFID and British High Commission (2002). LTU for VAT, implemented new VAT law in 2013 was also established to modernize the VAT system.

[12] Tax effort index is defined as the ratio of the actual tax share to the predicted (or potential) tax share. If the value of the index is less than one, it means that the country is not utilizing its full revenue potential. The predicted tax share is calculated by regressing the tax–GDP ratio on explanatory variables that serve as proxies for the tax base and other structural factors influencing tax revenue performance. For details of approaches toward measuring tax effort, see Stotsky and WoldeMariam (1997), Hudson and Tere (2003).

Higher tax efforts imply higher efficiency of the tax institution but it may be noted that there is a significant relationship between the higher efforts and countries' stages of development. Based on the tax effort index Bangladesh's performance is the lowest among some African and Asian countries (Appendix A Table 01).

Recent initiatives from Bangladesh government to modernize tax system kindle a flare of hope of moving ahead from the perspective of tax revenue effort, but lack of institutional capacity and absence of a neoteric forecasting method are hindering the enhancement of tax effort in Bangladesh. A relevant forecasting method to predict revenue components assists to maintain accuracy of fiscal estimates to avoid the risks which arise from imprudent policy settings leading to huge pressure on financing and sometimes it may bring erratic policy outcome.

IV. Methodology

IV. A. Data Description

To specify a model for the purpose of forecasting the tax revenue, monthly data of Tax revenue were collected from Ministry of Finance (MoF), Bangladesh. The collected dataset consists of a total of 101 observations from July 2004 to November 2012. Out of these 101 observations, 84 data points were used to specify a model and the remainders were used in this exercise to check for the fit of the specified model.

IV. B. Model Specification

There are two basic approaches to forecast time series: the self-projecting time series approach and the cause-and-effect approach. Cause and effect methods attempt forecasting based on underlying series, which are believed to cause the behaviour of the original series. The self-projecting time series uses only the time series data of the activity to generate forecasts. This latter approach typically requires far less data and is useful for short to medium-term forecasting. To analyze the self-projecting time series approach this exercise incorporates Holt–Winters seasonal multiplicative procedure, Holt–Winters seasonal additive procedure and Box-Jenkins methodology (ARIMA SARIMA multiplicative model) to forecast and evaluates the performance of these three procedures to find out the most appropriate one to forecast monthly tax revenue of Bangladesh.

SARIMA ARIMA Multiplicative Model

This exercise employs Box-Jenkin's methodology to forecast tax revenue using a seasonal ARIMA model, where the seasonal ARIMA process incorporates both non-seasonal and seasonal factors in a multiplicative model.

One shorthand notation for the model is ARIMA $(p, d, q) \times (P, D, Q, S)$, with p = non-seasonal AR order, d = non-seasonal differencing, q = non-seasonal MA order, P = seasonal AR order, D = seasonal differencing, Q = seasonal MA order, and S = time span of repeating seasonal pattern.

Without differencing operations, the model could be written more formally as

$$\Phi(B^S)\varphi(B)(x_t - \mu) = \Theta(B^S)\theta(B)w_t \tag{1}$$

The non-seasonal components are:

$$\text{AR: } \varphi(B) = 1 - \varphi_1 B - \ldots - \varphi_p B^p$$

$$\text{MA: } \theta(B) = 1 + \theta_1 B + \ldots + \theta_q B^q$$

The seasonal components are:

$$\text{Seasonal AR: } \Phi(B^S) = 1 - \Phi_1 B^S - \ldots - \Phi_P B^{PS}$$

$$\text{Seasonal MA: } \Theta(B^S) = 1 + \Theta_1 B^S + \ldots + \Theta_Q B^{QS}$$

A time series is said to follow an autoregressive (AR) model of order p if the current value of the series can be expressed as a linear function of the previous values of the series plus a random shock term. The general equation of an autoregressive model of order p, AR (p), can be written as

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + \ldots + a_p x_{t-p}$$

Where, $a_1, a_2, \ldots, a_p$ are the autoregressive model parameters.

And, the moving average (MA) model describes a time series that is a linear function of the current and previous random shocks (e). The random shocks are also called errors, residuals or a white noise process. A time series, x_t, is said to be a moving average process of order q, MA(q),

if,

$$x_t = e_t - b_1 e_{t-1} - b_2 e_{t-2} - \ldots - b_q e_{t-q}$$

Where, x_t is the current value of time series data; $e_t, e_{t-1}, \ldots, e_{t-q}$ the current and previous errors or random shocks; and $b_1, b_2, \ldots, b_q$ are the moving average model parameters.

Box-Jenkins forecasting models are based on statistical concepts and principles and are able to model a wide spectrum of time series behavior. It has a large class of models to choose from and a systematic approach for identifying the correct model form. There are both statistical tests for verifying model validity and statistical measures of forecast uncertainty. The underlying goal of this methodology is to find an appropriate formula so that the residuals are as small as possible and exhibit no pattern. The model- building process involves four steps. These steps are repeated as necessary; to end up with a specific formula that replicates the patterns in the series as closely as possible and produces accurate forecasts. These steps are-

i. Model identification and selection

ii. Parameter estimation

iii. Diagnostic checking

iv. Forecasting

In model identification and selection, covariance stationary data process is ensured first. A stochastic process y_t is covariance stationary if it satisfies the following requirements:

i. $E[y_t]$ is independent of t.

ii. $Var[y_t]$ is a finite, positive constant, independent of t.

iii. $Cov[y_t, y_s]$ is a finite function of t-s, but not of t or s.

To check for stationary series, this exercise intends to use both Augmented Dickey-Fuller and Phillips-Perron tests for unit root. The Dickey–Fuller test (developed by Dickey and Fuller in 1979) involves fitting the following model:

$$y_t = a + by_{t-1} + ct + ut \quad (2)$$

by an ordinary least squares, setting $a = 0$ if drift term is absent or $c = 0$ if trend term is absent . However, such a regression is likely to be plagued by serial correlation. To control for that, the augmented Dickey–Fuller test instead fits a model of the form, such as:

$$\Delta y_t = a + b_0 y_{t-1} + ct + b_1 \Delta y_{t-1} + b_2 \Delta y_{t-2} + ... + b_k \Delta y_{t-k} + e_t \quad (3)$$

Where, k is the number of lags specified. Testing $b_0 = 0$ is equivalent to testing $b = 0$, or, equivalently, y_t follows a unit root process.

The Phillips–Perron test involves fitting equation (2), and the results are used to calculate the test statistics. Phillips and Perron (1988) proposed two alternative statistics[13].

Using plots of the autocorrelation and partial autocorrelation functions of the dependent time series it is decided which (if any) autoregressive or moving average component should be used in the model to correctly specify the ARIMA SARIMA multiplicative model. Besides, parameter estimation uses computation algorithms to arrive at coefficients those best fit the selected ARIMA model. The most common methods use maximum likelihood estimation or non-linear least-squares estimation.

To perform diagnostic tests and to evaluate different combinations of autoregressive and moving average lags for both seasonal and non-seasonal portions, Akaike Information Criterion (AIC)[14] and Baysian Information Criterion (BIC)[15] are employed. Some authors define the AIC as the expression above divided by the sample size.

Portmanteau (Q) and Bartlett's periodogram-based tests for white noise are executed as diagnostic tools to explore whether the residuals are serially uncorrelated. The portmanteau test relies on the fact that if *x(1),...., x(n)* is a realization from a white-noise process. Then

$$Q = n(n+2)\sum_{j=1}^{m}\frac{1}{n-j}\hat{\rho}^2(j) \rightarrow \chi^2_m$$

Where, *m* is the number of autocorrelations calculated (equal to the number of lags specified) and indicates convergence in distribution to a χ^2 distribution with m degrees of freedom. $\hat{\rho}_j$ is the estimated autocorrelation for lag *j*.

Bartlett's periodogram-based test for white noise is a test of the null hypothesis that the data comes from a white-noise process of uncorrelated random variables having a constant mean and a constant variance.

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[13] Phillips and Perron's test statistics can be viewed as Dickey–Fuller statistics that have been made robust to serial correlation by using the Newey–West (1987) heteroskedasticity- and autocorrelation-consistent covariance matrix estimator.

[14] Akaike's (1974) information criterion is defined as : AIC = -2 lnL + 2k, where, lnL is the maximized log-likelihood of the model and k is the number of parameters estimated.

[15] Schwarz's (1978) Bayesian information criterion is another measure of fit defined as: BIC = -2 lnL + klnN here, N is the sample size.

If $x(1),\ldots, x(T)$ is a realization from a white-noise process with variance σ^2, the spectral distribution would be given by $F(w) = \sigma^2 w$ for $w \in [0,1]$, and we would expect the cumulative periodogram of the data to be close to the points $S_k = k/q$ for $q = [n/2]+1$; $k = 1,\ldots, q$. [n/2] is the greatest integer less than or equal to n/2. Except for w= 0 and w = .5, the random variables $2\hat{f}(w_k)/\sigma^2$ are asymptotically independently and identically distributed as χ_2^2. Because χ_2^2 is the same as twice a random variable distributed exponentially with mean 1, the cumulative periodogram has approximately the same distribution as the ordered values from a uniform (on the unit interval) distribution.

Feller (1948) shows that this results in

$$\lim_{q\to\infty} \Pr\left(\max_{1\le k\le q} \sqrt{q}\left|U_k - \frac{k}{q}\right| \le a\right) = \sum_{j=-\infty}^{\infty} (-1)^j e^{2a^2 j^2} = G(a)$$

Where, U_k is the ordered uniform quantile. The Bartlett statistic is computed as

$$B = \max_{1\le k\le q} \sqrt{\frac{n}{2}}\left|\widehat{F_k} - \frac{k}{q}\right|$$

Where, $\widehat{F_k}$ is the cumulative periodogram defined in terms of the sample spectral density $\hat{f}$ as,

$$\widehat{F_k} = \frac{\sum_{j=1}^{k} \hat{f}(w_j)}{\sum_{j=1}^{q} \hat{f}(w_j)}$$

The associated p-value for the Bartlett statistic and the confidence bands on the graph are computed as 1-G (B) using Feller's result. After the appropriate model is selected, the step of forecasting is executed on the ARIMA SARIMA multiplicative model using both one-step forecasting[16] and dynamic forecasting[17] methods.

Holt–Winters seasonal multiplicative procedure

This method forecasts seasonal time series in which the amplitude of the seasonal component grows with the series. Chatfield (2001) notes that there are some nonlinear state-space models whose optimal prediction equations correspond to the multiplicative Holt–Winters method.

[16] The one-step-ahead forecasts never deviate far from the observed values, though over time the dynamic forecasts have larger errors. As when making the one-step forecast for period t, we know the actual value of the data process xt at time t-1.

[17] Dynamic forecasting the forecasted value of xt for period t is based on the observed value of xt at period t-1 but the forecast for t+1 is based on the forecasted value at period t, the forecast for period t+2 is based on the forecasted value of period t+1 and so on.

This procedure is best applied to data that could be described by: $x_{t+j} = (u_t + B_j)S_{t+j} + e_{t+j}$, where x_t is the series, u_t is the time-varying mean at time *t*, B is a parameter, S_t is the seasonal component at time *t*, and e_t is an idiosyncratic error. There are three aspects to implementing the Holt–Winters seasonal multiplicative procedure: the forecasting equation, the initial values, and the updating equations. In this method, the data are now assumed seasonal with period *L*. Given the estimates *a(t), b(t), and s(t+m-L)*, a m step-ahead point forecast of x_t, denoted by$_{t+m}$, is $y^*_{t+m} = \{a(t) + b(t)m\}\ s(t+m-L)$, here, y^*_{t+m} denotes the estimated value of y_{t+m}. Given the smoothing parameters α, β, and γ, the updating equations are as follows:

$$a(t)=\alpha\{x_t/s(t-L)\}+(1-\alpha)\{a(t-1)+b(t-1)\},$$

$$b(t)=\beta\{a(t)-a(t-1)\}+(1-\beta)b(t-1)$$

and

$$s(t)=\gamma\{x_t/a(t)\}+(1-\gamma)s(t-L).$$

The updating equations require the *L+2* initial values *a(0), b(0), s(1-L), s(2-L),…., s(0)*. To calculate the initial values with the first n years, each of which contains *L* seasons. Here, *n* is set to the number of seasons in half the sample. The initial value of the trend component, *b(0)*, can be estimated as: $b(0)=(x^*_n-x^*_1)/(n-1)L$ where, x^*_n is the average level of x_t in year *n* and x^*_1 is the average level of x_t in the first year. The initial value for the linear term, *a(0)*, is then calculated as $a(0) = x^*_1-(L/2)b(0)$.

To calculate the initial values for the seasons *1, 2,…, L,* we first calculate the deviation-adjusted values, such as: $S(t)=x_t/[x^*_i-\{(L+1)/2-j\}b(0)]$, where, *i* is the year that corresponds to time *t*, *j* is the season that corresponds to time *t*, and x_i is the average level of x_t in year *i*.

Next, for each season $l = 1, 2,…,L$, we define p as the average S_t over the years. That is,

$$p = \left(\frac{1}{n}\right) \sum_{k=0}^{n-1} S_{l+kL}, for\ l=1, 2,..., L.$$

Then, the initial estimates are as follow:

$p_{0l} = p_l\left(\frac{L}{\sum_{l=1}^{L} p_l}\right)$, for l=1, 2,.., L, and these values are used to fill in *s(1-L),..., s(0)*.

Holt–Winters Seasonal Additive Method

In this method the seasonal effect is assumed to be additive rather than multiplicative. This method forecasts series that can be described by the equation: $x_{t+j} = (u_t + B_j) + S_{t+j} + e_{t+j}$, where x_t is the series, u_t is the time-varying mean at time t, B is a parameter, S_t is the seasonal component at time t, and e_t is an idiosyncratic error. As in the multiplicative case, there are three smoothing parameters, α, β and γ, which can either be set or chosen to minimize the in-sample sum-of-squared forecast errors. The updating equations are as:

$a(t)=\alpha\{x_t-s(t-L)\}+(1-\alpha)\{a(t-1)+b(t-1)\}$,

$b(t)=\beta\{a(t)-a(t-1)\}+(1-\beta)b(t-1)$

and $s(t)=\gamma\{x_t-a(t)\}+(1-\gamma)s(t-L)$.

An m-step-ahead forecast, denoted by y^*_{t+m} is given by $x^*_{t+m}=a(t)+b(t)m+s(t+m-L)$. To obtain the initial values for $a(0)$, $b(0)$, $s(1-L),\ldots, s(0)$ from the regression: $x_t=a(0)+b(0)t+d_{s,1-L}D_1+ d_{s,2-L}D_2+\ldots..+ d_{s,0}D_L+e_t$, where, $D_1,\ldots, D_L$ are dummy variables with $D_i=1$, if t corresponds to season i and $D_i=0$, otherwise.

Evaluation Criteria

In this exercise, the evaluation among the three approaches: Holt-Winters seasonal multiplicative model, Holt-Winters seasonal additive model and ARIMA SARIMA multiplicative model is performed applying Mean Percentage Error (MPE)[18], Mean Absolute Percentage Error (MAPE)[19] and Sum of Squared Residuals (SSR)[20].

V. Results and Discussions

Figure 1 represents the time series plot of monthly tax revenue collected by National Board of Revenue in Bangladesh and Figure 2 shows growth rate of tax revenue. From Figure 1, it is evident that tax revenue follows an increasing and deterministic trend along with seasonal patterns, i.e. every month of June in each fiscal year. Further, seasonality and trending pattern with the autocorrelation function of the data set.

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[18] MPE=$\frac{1}{n}\sum_{t=1}^{n}\frac{u_t}{x_t}$, here, $u_t = x_t - f_t$; the difference between the actual and forecasted values of x_t

[19] MAPE=$\frac{1}{n}\sum_{t=1}^{n}\left|\frac{u_t}{x_t}\right|$,here, $u_t = x_t - f_t$; the difference between the actual and forecasted values of x_t

[20] MSE=(1/n)$\sum_{t=1}^{n}u_t^2$, here, $u_t = x_t - f_t$; the difference between the actual and forecasted values of x_t

These seasonal variations correctly explain tax collections in Bangladesh as larger number of income tax and VAT returns are completed in June. The growth rate of revenue collection is quite volatile over the whole period of time showing upswings and downswings in it. Sometimes growth rate is positive and very high while other times growth rate is negative.

Figure 1: Trend of Total Tax Revenue (Crore Taka)

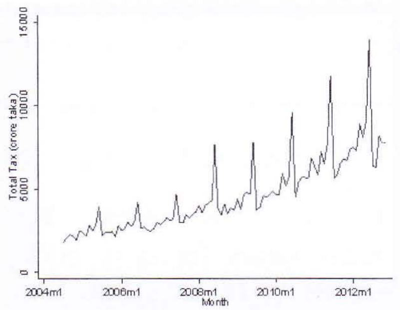
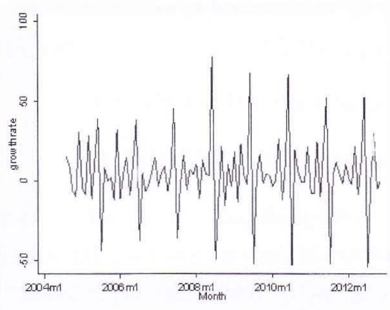


Figure 2: Growth Rate of Tax Revenue



Moreover, descriptive statistics of monthly tax revenue collection in Bangladesh is displayed in table 1, which exhibits some preliminary understanding about nature of the data. It shows that during 2011 total tax collection was the highest with mean value of 7827.32 crore taka. As indicated by Figure 1, total tax revenue has shown increasing trend with increasing variability between highest and lowest values within a year. Revenue collection is generally higher during April-June and lower during November-January. In terms of distributions, the series is also not normally distributed as the values of skewness and kurtosis is far from 0 and 3 respectively for the entire period.

Table 1: Descriptive Statistics of Tax Revenue in Bangladesh

Fiscal Year	Mean	Std. Deviation	Skewness	Kurtosis	Minimum	Maximum
2004-05	2442	568.8	1.55	5.15	1807	3947
2005-06	2703	557.2	1.70	5.59	2129	4214
2006-07	3014	595.3	1.98	6.53	2475	4700
2007-08	3952	1244.8	2.38	7.87	2961	7657
2008-09	4375	1181.1	2.21	7.24	3403	7813
2009-10	5167	1531.9	2.16	7.08	3729	9599
2010-11	6604	1845.4	1.89	6.27	4518	11762
2011-12	7827	2206.8	1.86	6.07	5621	13948
2012-13	7272	862.1	-.269	1.27	6378	8183

Source: Authors' Own Estimates

The autocorrelation and partial autocorrelation function are used to identify the nature of data; whether they follow any systematic pattern or not of the time series analysis. It also helps us to identify whether series is stationary or not. The ACF and PACF graph for tax revenue at level and first differences are represented in Figure 4a and 4b. Autocorrelation coefficient dies out slowly and statistically significant at seasonal interval for the series at 5% significance level. This indicates that the series is non-stationary and follows trend and seasonal pattern at level form.

Figure 3a: Detrended Tax Revenue

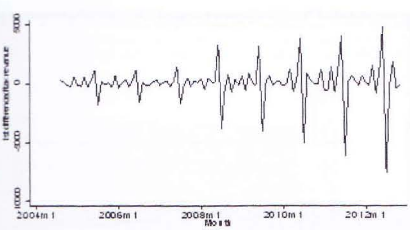


Figure 3b: Deseasonalised Detrended Tax Revenue

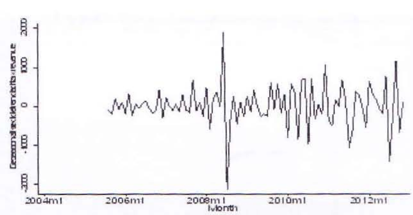
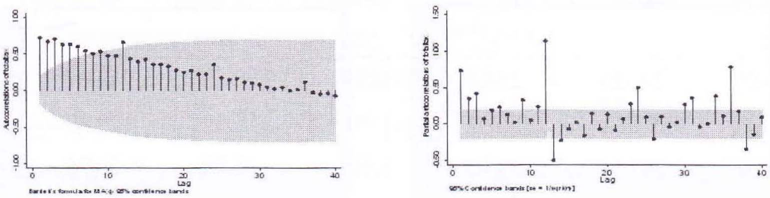


Figure 4a: ACF and PACF of Tax Revenue



While with first difference, the ACF of cement sales dies out quickly although at certain seasonal interval they are statistically significant at 5% significance level, indicating series is stationary. Similarly, a new series is generated from the first differenced series by considering seasonal difference of order s=12 (since it's a monthly data) and then ACF and PACF are obtained for the newly deseasonalised series. The result shown in Figure 4b indicates that new series is still non-stationary as ACF dies out slowly, but its first difference is stationary as represented in Figure 4c.

Figure 4b: ACF and PACF of Detrended Tax Revenue

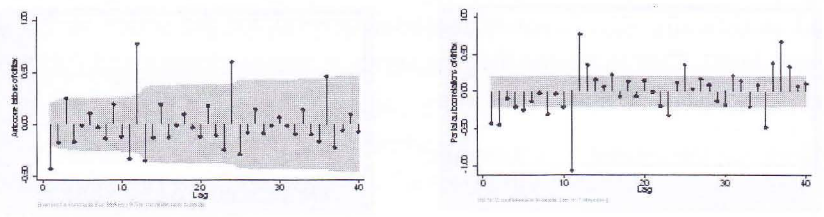
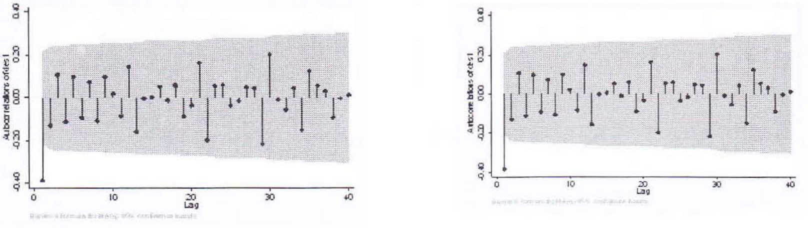


Figure 4c: ACF and PACF of Detrended Desesonalised Tax Revenue



To confirm the presence of non stationary in the data process Augmented Dickey-Fuller test (Table 2) for unit root is employed. Form Table 2, it can be concluded that there is no unit root present in the data process, as the value of test statistics is -6.731, smaller than the 1% critical value of -4.042, but the coefficient of trend term is 60.63 significant at even 1% level of significance. Therefore, tax revenue is non-stationary because of the presence of trend term. Detrending tax revenue through first difference ($\Delta x_t = x_t - x_{t-1}$) yields a stationary data process with a statistically insignificant trend coefficient (p-value more than 0.75). Deseasonalized detrended tax revenue is also stationary with test statistics of -10.66 and statistically insignificant trend term.

Like Augmented Dickey-Fuller test, Phillips-Perron test for unit root (Table 3) concludes the presence of non-stationarity; not because of unit root (value of test statistics is -8.88 lower than 1% critical value of -4.04) but because of the presence of statistically significant trend coefficient. First difference yields detrended and stationary data process with no unit root and statistically significant trend coefficient.

Table 2: Augmented Dickey-Fuller Test for Unit Root

Variable	Test statistics z(t)	1% critical value	L1	Trend	Constant	Lag. difference(1)
Tax revenue	-6.731	-4.042	-.9236*	60.6278*	1253.453*	.0272**
1st differenced tax revenue	-13.365	-4.044	-2.083*	.8325**	79.580**	.4523*
Deseasonalised 1st differenced tax revenue	-10.661	-4.071	-1.856*	-.3799**	36.21**	.3374*

* With a p-value of less than 0.003. ** With a p-value of more than 0.75.

Table 3: Phillips-Perron Test for Unit Root

Variable	Test statistics z(t)	1% critical value	L1	Trend	Constant
Tax revenue	-8.875	-4.040	.1025#	58.561*	1245.237*
1st differenced tax revenue	-18.930	-4.042	-.4343*	.3754**	64.4123**
Deseasonalised 1st differenced tax revenue	-15.423	-4.069	-.3879*	-.2649**	22.3844**

* With a p-value of 0.000. ** With a p-value of more than 0.75. # With a p-value of more than 0.30

To determine the lags of ARIMA SARIMA multiplicative model; a close inspection of autocorrelation and partial autocorrelation functions of stationary first differenced tax revenue in Figure 4b and of autocorrelation and partial autocorrelation of detrended deseasonalised tax revenue in Figure 4c is conducted. It can be seen that, first lag of ACF and first two lags of PACF for detrended tax revenue are statistically significant as they falls within the 95% confidence interval other higher order statistically significant lags depict seasonal patterns. And for detrended deseasonalised tax revenue first lag of ACF and PACF falls within the 95% confidence interval.

Table 4: Regression Table with Diagnostic Tests

Model	Coefficients				AIC	BIC	Portmanteau (Q) test for white noise p>chi²(40)	Bartlett's Periodogram based test for white noise (p>B)
	Non-seasonal		Seasonal					
ARIMA.SARIMA (0,1,1)*(1,0,0,12)	MA(1)	-6.6	AR(1)	.934	1272.2	1279.5	0.2024	0.9795
ARIMA.SARIMA (1,1,1)*(1,0,0,12)	AR(1) MA(1)	.024* -.672	AR(1)	.935	1274.2	1283.2	0.2068	0.9798
ARIMA.SARIMA (0,1,2)*(1,0,0,12)	MA(1) MA(2)	-.649 -.02*	AR(1)	.935	1274.2	1283.9	0.2075	0.9801
ARIMA.SARIMA (2,1,1)*(1,0,1,12)	AR(1) AR(2) MA(1)	-.034* -.59* -.66	AR(1) MA(1)	.937 -.0002*	1295.4	1310.0	0.2699	0.9799

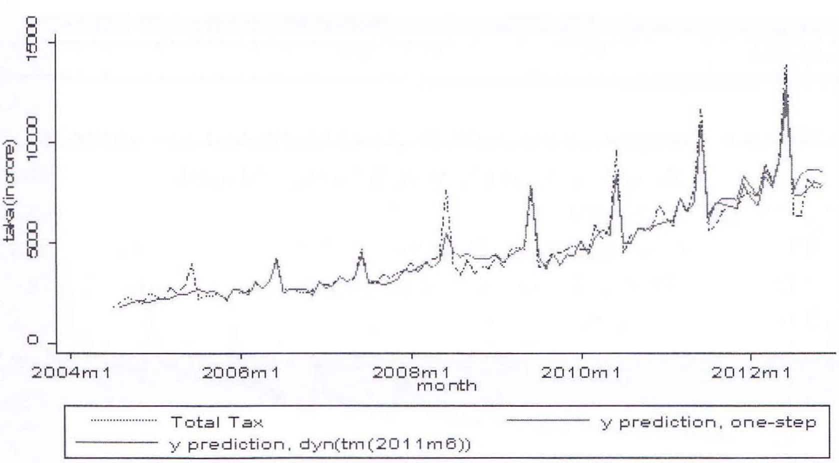
*All coefficient values are statistically significant with a p-value of 0.000 except for the coefficients with asterisk mark.

Estimation results of ARIMA SARIMA multiplicative model with different lags of autoregressive and moving average process for both seasonal and non-seasonal portion are shown in Table 4 with the results of diagnostic tests. Though for all the models are weak from the perspective of Portmanteau test for white noise, according to Bartlett's periodogram based test for white noise, all of them perform satisfactorily; error terms falling inside 95 percent confidence interval band (Appendix C Figure 03). Using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC); ARIMA (0,1,1) SARIMA(1,0,0,12) can be chosen as the most appropriate ARIMA SARIMA multiplicative model with the lowest AIC and BIC values of 1272.27 and 1279.53 respectively. All the coefficients of the lags of autoregressive and moving average for both seasonal and non-seasonal portions of this model are statistically significant with a p-value of 0.000.

After choosing the appropriate model, this exercise attempts to determine the forecasted value for total NBR tax revenue from July 2004 to November 2012 using both one step and dynamic forecasting methods in Figure 5.

Figure 5: Comparison among Actual and Forecasted Tax Revenue

(Based on Multiplicative SARIMA Model)



It is evident from the figure that, the estimated forecasting model assigns quite accurate approximation of the actual data. Though dynamic forecast deviates more than one step forecast, this deviation is not alarming and is somewhat expected as dynamic forecast uses estimated values to forecast.

Only abnormal discrepancy in one-step forecasting occurred in the month of June 2008. During this period, huge surplus in income tax had contributed a larger share in this surplus in revenue as interim care taker government of Bangladesh compelled many taxpayers to return due tax payments.

For both Holt-Winters seasonal multiplicative approach and Holt-Winters seasonal additive approach smoothing parameters α, β and γ are chosen to minimize the in-sample penalized forecast error. For multiplicative approach they are respectively 0.2571, 0.1271 and 0.3290. On the other hand, for additive approach they are 0.2179, 0.0550 and 1.000 respectively. Root mean squared error from multiplicative approach is 323.3492, smaller than that of additive approach, which is 407.1164. Therefore, it can be concluded that seasonal component grows with the series rather than being constant over the period.

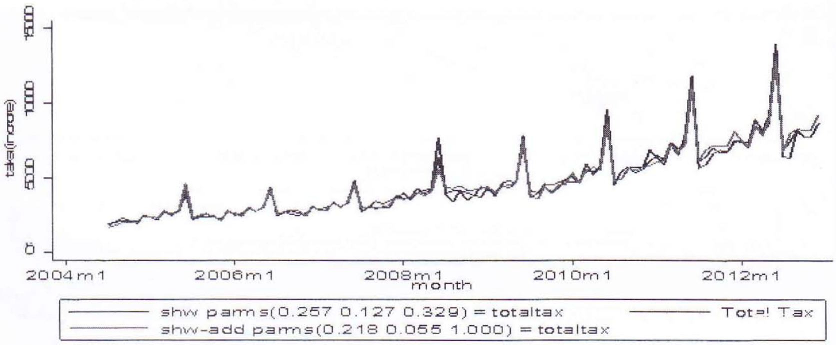
Table 5: Holt Winter's Multiplicative Approach

Alpha(α)	0.2571
Beta(β)	0.1271
Gamma(γ)	0.329
Sum of squared residuals	8887151
Root mean squared error	323.3492

Table 6: Holt Winter's Additive Approach

Alpha(α)	0.2179
Beta(β)	0.055
Gamma(γ)	1
Sum of squared residuals	1.41E+07
Root mean squared error	407.1164

Figure 6: Comparison among Actual and Forecasted Tax Revenue (Based on Multiplicative SARIMA Model)



VI. Evaluation of the Forecasting Model

In this section, this study evaluates various forecasting models based on the accuracy in tax revenue forecasting. Accuracy has been measured in terms of tax gaps, i.e. the difference between actual and projected values. Thus, minimum gap, either in the form of positive (actual tax revenue is above the projected value) or negative (actual tax revenue is below the projected value) indicates more accuracy in forecasting method. Higher volatility in tax gaps executing from a specific forecasting methodology leads to higher inaccuracy to project tax revenue, which may give birth to miss-match in a stable fiscal milieu.

Table 7: Statistical Measures of Accuracy of the Methods

Statistical methods	Mean Percentage Error (MPE)	Mean Absolute Percentage Error (MAPE)	Mean Square Error (MSE)
One-Step Forecast(ARIMA SARIMA)	-0.028	7.04	4409650
Dynamic Forecast (ARIMA SARIMA)	-0.076	9.68	505824.6
Holt-winter seasonal multiplicative approach	-0.015	4.38	96525.72
Holt-winter seasonal additive approach	-0.045	7.16	270845.5

Out of four competing methods, Holt-Winter Seasonal Multiplicative Approach performs the best in terms of minimum MSE and MAPE criteria because values of MAPE and MSE are lower compared to any forecasting method scrutinized in this study. From the three remaining methods, One-Step ARIMA SARIMA method can be ranked second. The MPE and MAPE from one-step ARIMA SARIMA approach are -0.028 and 7.04 respectively whereas from Holt-Winter multiplicative they are -0.015 and 4.38 respectively.

Table 8: Actual and Projected Tax Revenue

Fiscal Year	Actual	MoF Projection	One-step (ARIMA SARIMA)	Dynamic (ARIMA SARIMA)	Holt Winter (Multiplicative)	Holt Winter (Additive)
2004-05	293	322	240	240	295	294
2005-06	325	357	332	332	329	328
2006-07	362	411	358	358	363	363
2007-08	474	439	435	435	449	436
2008-09	525	545	560	560	535	554
2009-10	620	610	594	594	607	601
2010-11	793	726	770	770	783	764
2011-12	939	919	939	960	934	950
2012-13	364	1123	384	419	379	394

Source: Authors' Own Estimates except for MoF projection

Table 8 reports actual and forecasted tax revenue from fiscal year 2004-05 to 2012-13, whereas, MoF projected tax revenue has been taken from the budget documents reported by Ministry of Finance, Bangladesh. From these reported tax revenue, it is explicit that forecasted values of tax revenue generated from Holt-Winter multiplicative approach are closer to actual tax revenue from fiscal year 2004-05 to 2012-13 than other methods. Looking forward to Table 9, in which all reported values are in differenced form of actual and projected tax revenue values.

Table 9: Comparison among the Projections from Different Tax Revenue Forecasting Approaches

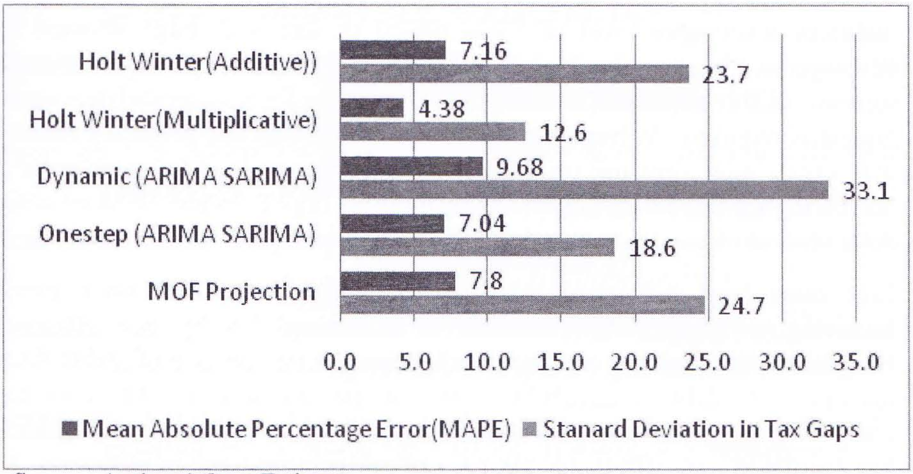
Fiscal Year	MoF Projection	One-step (ARIMA SARIMA)	Dynamic (ARIMA SARIMA)	Holt Winter Multiplicative Approach	Holt Winter Additive Approach
2004-05	-22.5	35.2	35.2	-0.4	-2.3
2005-06	-15.9	-8.0	-8.0	-4.8	-4.0
2006-07	-38.4	3.6	3.6	-1.0	-1.7
2007-08	21.3	39.8	39.8	25.5	37.9
2008-09	-20.0	-35.0	-35.0	-10.5	-28.9
2009-10	-12.6	25.6	25.6	12.7	18.9
2010-11	37.3	22.6	22.6	9.4	28.1
2011-12	13.3	0.3	-20.8	5.7	-11.2
2012-13	-21.2	-20.3	-55.3	-15.6	-30.3

Source: Authors' Own Estimates, but, MoF Projection is based on tax revenue data reported by Ministry of Finance, Bangladesh.

Tax gaps may either be positive or negative. Negative values in Table 9 indicate tax revenue in shortfall, but positive values indicate tax revenue in surplus. Moreover, model accuracy is appropriate or not, depends on the lower values of negative or positive, which indicate the gap between actual and projected tax revenue. Table 9 shows that amount of tax gap (forecasting error) is lower in case of Holt-Winter Multiplicative approach.

Further inspection on the association between standard deviation of tax gaps and the Minimum Absolute Percentage Error (MAPE) can show the reason why, Holt-Winter multiplicative method can be employed as an appropriate model to forecast tax revenue in Bangladesh.

Figure 7: Tax Gap Volatility among Various Forecasting Methods



Source: Authors' Own Estimates.

Here, Figure 7 shows that both standard deviations in tax gap and MAPE for different approaches which are the highest from judgmental projection by Ministry of Finance and the lowest from Holt-Winter multiplicative method. This indicates that lower value of MAPE is a good indicator of lower value of standard deviation of tax gaps. Thus, Holt-Winter multiplicative approach leads to more efficiency with minimum error to forecast tax revenue in Bangladesh comparing to the judgmental approach done by Ministry of Finance.

Vii. Conclusion and Policy Suggestion

Selecting an appropriate forecasting method to predict the nature of any macroeconomic series can be difficult because sometimes, any unforeseen event i.e. sudden crisis in world and domestic economy can change the whole calculation of forecasted value. This is why, forecasting of any series is a continuous process rather a one-time calculation.

Government revenue forecasting is an important aspect in the design and execution of sound fiscal policies. Moreover, it is important to enhance domestic resource mobilization, and to reduce heavy reliance on external financing. Because of liquid constraint and European debt crisis, external financing from multi-agency has started to become scarce; that is why, most of the least developed countries are compelled to seek alternative sources of deficit financing resulting in the gradual increase in domestic debt. To maintain sustainable level of fiscal deficit in line with huge demand for development budget for higher economic growth without the alarming increase in domestic debt; accuracy in tax revenue forecast in each upcoming budget is required. As huge difference between actual and projected revenue may create huge pressure on domestic financing and lead to larger scale of borrowing from the banking system resulting in high inflation, reduced actual development expenditure.

Like other least developed countries, Bangladesh's reliance on domestic financing to mitigate fiscal deficit is increasing day by day. Although Bangladesh is operating relatively well from the perspective of stable fiscal management, debt sustainability; revenue performance is still very low comparing to similar countries around the globe. In recent years, Bangladesh has increased tax efforts by taking various reforms and modernizing tax system. Nevertheless, it is not good enough because of the huge difference between actual and projected tax revenue reported by Ministry of Finance. This implies that judgmental tax revenue forecasts reported by Ministry of Finance may not follow an appropriate forecasting procedure. Therefore, this is high time to reduce tax revenue forecasting error, which follows an erratic trend - accounts for huge surpluses or huge shortfalls in tax revenue collection - over the couple of years.

The main objective of this study is to identify an appropriate methodology to forecast monthly tax revenue. This paper utilizes monthly tax revenue series from July 2004 to December 2012. Out of the four popular techniques scrutinized in this study, Holt-Winter multiplicative approach is found to be appropriate for revenue forecasting in Bangladesh with minimum forecast error.

Moreover, this study attempts to compare forecast tax revenue error projected by Ministry of Finance with others popular technique and finds that, existing judgmental tax revenue forecasting method in Bangladesh produces larger error with higher volatility in tax gaps compared to others popular methods employed in this study whereas, Holt-Winter multiplicative approach performs the best to forecast monthly tax revenue in Bangladesh. Thus, forecasting attempts in this paper have opened an avenue for the systematic analysis of revenue forecasting using several methods rather than depending on existing judgmental method followed by fiscal Authority in Bangladesh.

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Appendix A: Tables

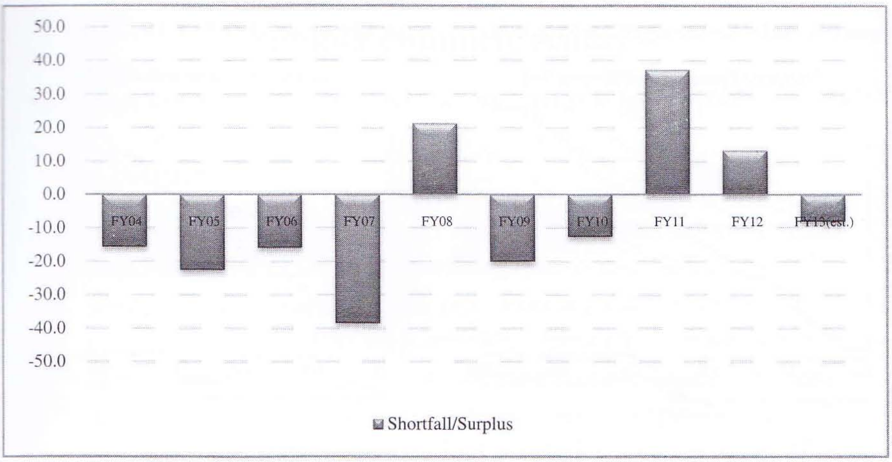
Table 01: Tax Efforts in Selected Countries

Name of Countries	Total Tax	Income Tax	Value Added Tax
Bangladesh	0.508	0.357	0.482
Bhutan	0.661	0.979	0.612
Nepal	0.736	0.452	0.764
India	0.775	0.975	0.774
Pakistan	0.942	0.952	0.969
Sri Lanka	1.182	0.677	1.812
Indonesia	1.014	1.283	0.821
Philippines	1.020	1.190	0.743
Singapore	1.009	0.967	0.989
South Korea	1.206	1.372	1.570
Thailand	0.936	0.705	0.687
Kenya	1.367	1.984	1.492
Tunisia	1.516	1.368	1.492
Uganda	0.947	0.877	1.286

Source: An Evaluation of the Tax System in Bangladesh- IGC Working Paper Series.

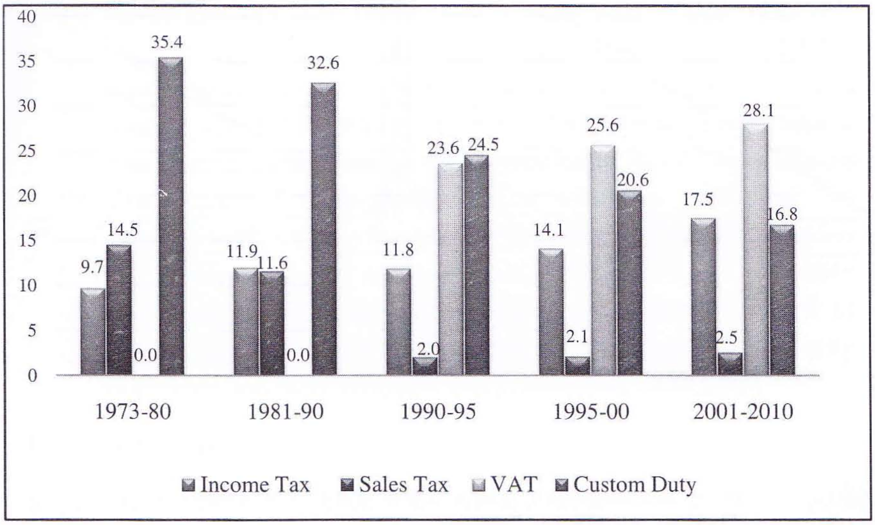
Appendix B: Figures

Figure 01: Shortfall or Surplus of Tax Revenue



Source: National Board of Revenue & Authors' Own estimates

Figure 02: Bangladesh: Tax Revenue Structure From 1972-2010



Source: Ministry of Finance, Bangladesh

Figure 03: Bartlett's Periodogram Based Test for White Noise

Figure 03a: ARIMA.SARIMA (0,1,1)*(1,0,0,12)

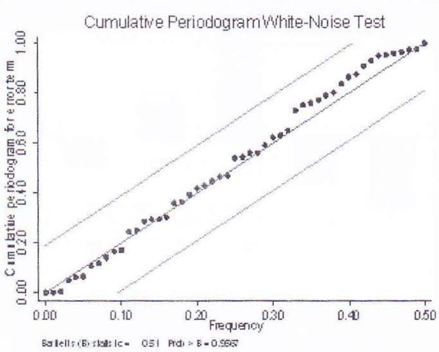


Figure 03b: ARIMA.SARIMA (1,1,1)*(1,0,0,12

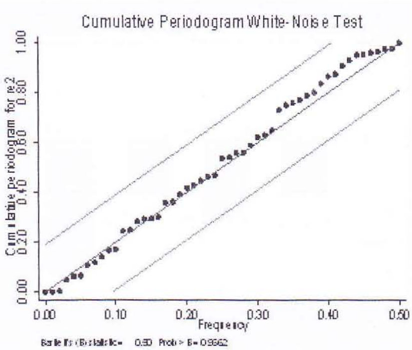


Figure 03c: ARIMA.SARIMA (0,1,2)*(1,0,0,12)

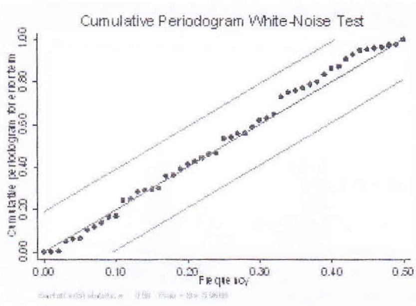


Figure 03d: ARIMA.SARIMA (2,1,1)*(1,0,1,12